Asset Pricing with Capital-Skill Complementarities*

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Abstract

I study a general equilibrium production model with capital-skill complementarities and exogenous limited participation to rationalize the decline in the risk-free rate, increase in equity risk premium, and increase in stock market participation over the last 30 to 40 years. I show that increasing reliance on high-skilled labor can account for some of these trends. As the economy depends more on high-skilled labor, the overall wage bill becomes smoother and smaller, which produces more volatile dividends. However, the wage bill to high-skilled workers becomes more volatile due to increasing capital-skill complementarities. As a result, high-skilled workers' labor income risk increase. Since high-skilled workers are the only participants in financial markets in the model, their stochastic discount factor becomes more volatile as a result of an increase in their labor income risk, which decreases the risk-free rate and further increases the equity risk premium.

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1 Introduction

Over the last 30 to 40 years, risk-free rates have experienced a secular decline while the risk premium on risky assets have increased.[1] In particular, the equity risk premium has also increased. Figure 1 shows that in 1979 to 1999 the average one-year ahead equity risk premium is 5.7% but in 2000 to 2020 it is 8.5%.[2] However, in the same time period stock market participation has generally increased, from 20.7% in 1983 to 29.5% in 2019 as shown in Figure 2. Nominal stock market wealth per household has also increased by more an order of magnitude, as shown in Figure 3. Standard theory predicts that if participation in the stock market increases then the equity risk premium declines and the risk-free rate increases.[3] The empirical trends, however, contradict these predictions. In this paper, I reconcile these facts by building a general equilibrium production-based asset pricing model in which increasing participation is driven by high-skilled workers. The production function also features an increase in dependence on high-skilled labor in production, which leads to an increase in high-skilled labor income risk and the riskiness of dividend cashflows.[4] The model, is therefore, able to match several of these trends.

Figure 2 also plots the participation rate in the stock market by college attainment. The increase in stock market participation is not equal; it is driven by individuals and households who are college educated. The participation rate for college graduates increases from 37.5% to 48.8%; for non-college it is 15.6% to 18.4%. College graduates also hold substantially more wealth in the stock market as shown in Figure 3. Many of the standard theories assume that agents all face the same background risk and only differ exogenously in terms of participation in financial markets.[5] However, empirically non-college graduates and college graduates fact very different labor income risks.

Differences in human capital and labor income risks play a significant role in determining asset prices. Krusell, Ohanian, Ríos-Rull, and Violante (2000) estimate that high-skilled workers are gross complements to capital whereas low-skilled workers are gross substitutes to capital. They use these estimates to show how the skilled wage premium can increase while the relative supply of high-skilled workers increase. Furthermore, Braxton, Herkenhoff, Rothbaum, and Schmidt (2021) show that while aggregate labor income risk has slightly declined, the labor income risk of high-skilled workers or college graduates has increased. Kogan, Papanikolaou, Schmidt, and Seegmiller (2022) attribute part of this increase in labor income risk to the fact that recent labor displacing

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[1] Caballero, Farhi, and Gourinchas (2017) use a modified version of the measure constructed by Gomme, Ravikumar, and Rupert (2011) to show the spread between the real return to capital and real risk-free rate. Gormsen and Huber (2022) and Crouzet and Eberly (2023) also document an increase in the risk premium in a similar time period.

[2] Caballero, Farhi, and Gourinchas (2017) and Farhi and Gourio (2018) also document an increase in the equity risk premium in a similar time period.

[3] See Heaton and Lucas (1996), Basak and Cuoco (1998), Brav, Constantinides, and Geczy (2002), and Vissing-Jørgensen (2002) among others for a detailed discussion on the effects of incomplete markets and increased participation.

[4] From here on, I use high-skilled/low-skilled and college/non-college graduate interchangeably.

[5] There are papers that do not assume this. For example, Danthine and Donaldson (2002) and Berk and Walden (2013) have participants and non-participants facing different background risks in the form of different labor income risks.

technology has affected high-skilled workers more. Finally, over the last 30 to 40 years the economy has grown more reliant on high-skilled workers. Figures 4 and 5 show the employment share and labor share by education, respectively. There is a secular increase in both for college graduates.

I build a dynamic general equilibrium model that features high-skilled and low-skilled households along with a production technology with capital-skill complementarities in the spirit of Krusell et al. (2000) to link these various facts and trends. I use capital-skill complementarities to generate different labor income risks between high-skilled and low-skilled households. I also assume that only high-skilled households have access to financial markets and low-skilled households are constrained or hand-to-mouth. While this assumption is stark, there is some justification in the data and this allows the model to be more tractable. This assumption implies that only the high-skilled household's stochastic discount factor (SDF) matters for asset pricing implications. Thus, factors that change the risk dynamics of high-skilled labor income feed into the behavior of asset prices and risk premia.

To examine these trends with this model, I calibrate the model to reflect three periods. The benchmark calibration that targets the entire time series in the data (1979 to 2020). There are two alternative calibrations that target the first and second half of the time series. I call the calibration that targets the first half the first alternative calibration and the calibration that targets the second half the second alternative calibration. Taken together, moving from the first alternative to the benchmark to the second alternative reflects the changing technological dependence and inputs over time. I target the labor shares by skill and the skilled wage premium, among other macroeconomic moments and trends, to reflect the changing dependence on and quantity of high-skilled labor. This strategy allows me to examine how these macroeconomic trends are related to the trends of financial market participation, the risk-free rate, and risk premia. I also solve and calibrate a simple model, which features homogenous labor and households only differ in their participation in financial markets. I compare and contrast the results of the simple model with my model, which I also refer to as the baseline model.

Both models are able to replicate the standard macroeconomic moments; by construction the simple model cannot generate moments by skill type. Furthermore, both models are able to produce a sizeable equity risk premium and low and stable risk-free rate. Though, the benchmark calibration of my model can generate a slightly larger equity risk premium. The largest departure of performance, however, are the models' abilities to generate the observed macroeconomic and financial trends. While the simple model generates a smaller risk premium and higher risk-free rate as participation increases, my model is able to generate the opposite, which is what is observed in the data. From the first alternative calibration to the second alternative calibration the equity risk premium increases by 0.493 percentage points or a 12.9% increase. This is driven by the risk-free rate declining 0.117 percentage points (a 6.7% decrease) and equity returns increasing 0.376 percentage points (a 6.7% increase).

The baseline model generates these outcomes through several channels that offset the effects of increasing financial market participation. The first channel is through the shift from low-skilled labor to high-skilled labor that induces a change in the composition of the overall wage bill. Since high-skilled labor is more complementary to capital than low-skilled labor, the high-skilled wage bill is smoother than the low-skilled wage bill. Therefore, the shift to high-skilled labor makes the wage bill smoother, and thus the wage bill a less effective countercyclical force on profits and dividends. Furthermore, model generates a declining overall labor share. The model generates a lower overall labor share through the greater dependence on high-skilled labor in production and increasing proportion of high-skilled labor. Since the overall wage bill still exerts countercyclical pressure on dividends, making it relatively smaller leads to dividends becoming more volatile.

These channels not only operate through the production side of the economy but also through the consumer's side through a general equilibrium response. While the overall wage bill becomes smoother, the high-skilled wage bill becomes more volatile. This is driven by the increasing capital-skill complementary channel. The low-skilled wage bill is still more volatile than the high-skilled wage bill and the compositional shift offsets the increase in the high-skill wage bill volatility to make the overall wage bill smoother. However, high-skilled households experience an increase in labor income risk and are the only participants in financial markets. In equilibrium this is a major source of the increase in the consumption volatility of high-skilled households. This increases the volatility of their SDF and since they are the only participants in financial markets, this channel contributes to a lower risk-free rate and higher equity risk premium. The increased volatility of dividends also features a similar general equilibrium effect, but this channel is present even in asset pricing models that have a trivial production sector. The increase in labor income volatility and dividend volatility offset the effect of increased participation.

This paper is related to several literatures. The first is the literature studying production-based general equilibrium asset pricing models. These models endogenously determine both the SDF and dividend process. This modelling approach integrates asset pricing implications into a more general macroeconomic setting. These models exacerbate issues such as the equity risk premium since endogenizing both the SDF and dividend process leads to excessive smoothing. Jermann (1998) and Boldrin, Christiano, and Fisher (2001) are among the first to overcome this issue; both papers combine habits persistence in the spirit of Campbell and Cochrane (1999) and capital adjustment costs to produce a low-risk free rate and sizeable equity premium. Production-based general equilibrium models have since been used to study a wide range of problems. Examples include: Fama and Schwert (1977), Danthine and Donaldson (2002), Gomes, Kogan, and Zhang (2003), Uhlig (2007), Papanikolaou (2011), Gourio (2012), Gourio (2013), Ai and Kiku (2013), Kogan and Papanikolaou (2014), Bai, Hou, Kung, Li, and Zhang (2019), and Kogan, Papanikolaou, and Stoffman (2020), among others. Danthine and Donaldson (2002) is the most related to this paper. They show that firms insure workers, who have restricted access to financial markets, which generates operating leverage and in turn a sizeable risk premium on equity claims. The firm's stochastic discount factor is solely determined by firm owners who do not provide labor. My model differs as high-skilled workers are also the firms' owners. Furthermore, I focus on how changing technology and inputs affect risk premium dynamics. In particular, I examine how changing technology and inputs changes the nature of the wage bill and how that feeds into asset pricing implications.

Recently, several general equilibrium production models feature endogenous long-run consumption and dividend risk. Kaltenbrunner and Lochstoer (2010) and Croce (2014) show how to integrate the insights of Bansal and Yaron (2004) into this setting through long-run aggregate productivity growth risk. These models can produce a sizeable risk premium as well as low and stable risk-free rates while employing Epstein and Zin (1989) and Weil (1990) preferences. Examples of papers that employ this mechanism include: Ai, Croce, and Li (2013), Kung and Schmid (2015), Favilukis and Lin (2016b), Ai, Croce, Diercks, and Li (2018), and Favilukis, Lin, and Zhao (2020), among others. This work is most related to Favilukis and Lin (2016b) and Favilukis, Lin, and Zhao (2020). Similar to Danthine and Donaldson (2002), Favilukis and Lin (2016b) and Favilukis, Lin, and Zhao (2020) incorporate operating leverage in the wage bill. They accomplish this through wage rigidity rather than firms insuring workers, however. Favilukis and Lin (2016b) study how real wage rigidity can improve the quantitative performance of general equilibrium asset pricing models and Favilukis, Lin, and Zhao (2020) show how labor market frictions affect credit markets. The wage bill in my model becomes smoother through changing technology and inputs rather than through wage rigidity.

Another area of research this paper is related is the growing literature in macroeconomics and finance that seeks to jointly explain macroeconomic and financial trends such as the declining labor share, lackluster investment, declining risk-free rates, risk premia, and high valuations. Examples of this literature include Farhi and Gourio (2018), Hartman-Glaser, Lustig, and Xiaolan (2019), Barkai (2020), Covarrubias, Gutiérrez, and Philippon (2020), Corhay, Kung, and Schmid (2020), De Loecker, Eeckhout, and Unger (2020), and Seegmiller (2023) among others. My paper examines the most similar set of trends as Farhi and Gourio (2018). They attribute a decreasing risk-free rate, stable risky returns, declining labor share, lackluster investment, and stable valuation ratios to rising market power, rising unmeasured intangibles, and rising risk premia. Farhi and Gourio (2018) show this in a model that features time-varying disaster risk in the spirit of Gabaix (2011, 2012) and Wachter (2013). My approach differs from Farhi and Gourio (2018) as I employ long-run risk in productivity growth instead of a time-varying rare disaster risk. Furthermore, I complement this literature by also incorporating the trends related to increased financial market participation and high-skilled labor such as the increase in proportion of high-skilled labor, the increase in the high-skilled labor share, and increase and recent stabilization of the skilled wage premium.

Finally, this paper is related to is the emerging literature at the intersection of financial economics and labor economics. Human capital is a significant source of wealth for most individuals and labor is among the largest and most expensive inputs for firms' production. As a result, labor market frictions and the uninsurability of human capital have important implications for both financial economics and labor economics. This literature includes Matsa (2010), Eisfeldt and Papanikolaou (2013), Berk and Walden (2013), Donangelo (2014), Belo, Lin, and Bazdresch (2014), Palacios (2015), Belo, Li, Lin, and Zhao (2017), Kuehn, Simutin, and Wang (2017), Favilukis and Lin (2016a,b), Donangelo, Gourio, Kehrig, and Palacios (2019), Zhang (2019), Favilukis, Lin, and Zhao (2020), Kogan, Papanikolaou, Schmidt, and Song (2020), Petrosky-Nadeau, Zhang, and Kuehn (2018), Seegmiller (2023), Donangelo (2021), and Kogan et al. (2022) among others. I contribute to this literature by investigating the implications of changing capital-skill complementarities and increasing reliance on high-skilled labor in production for asset pricing.

I organize the remainder of the paper as follows. Section 2 describes the data sources and variable construction. Section 3 presents the model. Section 4 discusses the calibration strategy. Section 5 analyzes the model's results. Finally, Section 6 concludes.

2 Data

This section briefly describes the datasets used in this study as well as the steps taken to produce certain key variables or estimates. I utilize various datasets ranging from macroeconomic and financial time series as well as publicly available microdata. My empirical analysis and data are limited to the United States. More precise details on the data and the cleaning/merging process can be found in Appendix B.

I get the time series on various macroeconomic series such as aggregate consumption, GDP, etc. from the National Income and Product Accounts (NIPA) tables, which is maintained by the Bureau of Economic Analysis (BEA). NIPA Table 1.1.5 provides GDP and its expenditure components. I use following line items from this table: GDP, personal consumption expenditures, gross private domestic investment (including its subcomponents such as residential and nonresidential investment), and government expenditures. I also utilize NIPA Table 2.1 which contains information on aggregate personal income. Furthermore, I take the line items wages and salaries to private industries and supplements to wages and salaries to produce an estimate of the total wage bill for the private sector. I use the quarterly frequency series from 1947:Q1 to 2021:Q4 for both tables.

Next, I use the time series for the total private sector employment and for the CPI from the Bureau of Labor Statistics (BLS). Both datasets are taken at the quarterly frequency by averaging over the monthly frequency series. Converting quarterly data to annual data follows the same procedure. The time range for the data on private sector employment is January 1939 to December 2021 and for the CPI it ranges from January 1913 to December 2021. All nominal variables are deflated using the CPI.

I also use the Consumer Population Survey (CPS), which is a monthly survey of households conducted by the BLS that contains information such as education, earnings, employment status etc. The NBER provides annual extracts of these files called the Merged Outgoing Rotation Groups (MORG). The data are available from 1979 to 2020. The combination of the CPS data and standard macroeconomic time series allows me to create estimates of both employment and wage bill/labor share by education. I use the CPS data to create the shares of employment and the wage bill/labor share by education.

To obtain these estimates, first I keep only individuals employed by the private sector since the model only features a private sector. Next, I identify who is a high-skilled or college educated worker; an individual is considered to be college educated if they have completed at least 16 years of education. The CPS provides data on the highest grade attended and whether the highest grade was completed. If the individual has completed the highest grade attended that is the number of years of education, if not then the number of years of education is coded as the highest grade attended minus one. Then by the year and quarter, I take a weighted sum of the weekly earnings by education. The CPS provides weights for aggregation so that aggregate statistics are reflective of the U.S. population aged 16 years and older. The ratio of the weighted sum of weekly earnings by education gives an estimate of the share of the wage bill going to high-skilled/low-skilled workers. An estimate of the share of total employment by skill is obtained in a similar manner with these weights. These estimates combined with the relevant line items from the NIPA Tables allow me to construct total employment and wage bill/labor share by education.

Next, I use the Survey of Consumer Finances, published by the Federal Reserve to obtain household portfolio data. The survey is published triennially, and I use the surveys from 1983 to 2019. I obtain data for household level holdings of debt, equities, and liquid assets (such as cash, checking accounts, U.S. government savings bonds etc.). For stock market holdings specifically, I define the total household holdings of stock as the sum of direct holdings, indirect holdings (investment in mutual funds that hold stock), and holdings in tax-advantaged retirement accounts (Roth IRAs, 401(k) plans etc.). Households are defined are participating in the stock market if they hold strictly positive wealth in the stock market. More details in the construction of these variables can be found in Appendix B.

Finally, I describe the datasets for the stock market return series and firm characteristics. The Center for Research in Security Prices (CRSP) and Kenneth French's website provide U.S. stock market returns. These files contain proxies for the total equity return and risk-free rate at the monthly frequency. The data ranges from 1929 to 2022. Compustat Fundamental Quarterly files from March 1961 to September 2022 are used to construct profit shares. I only keep companies that are based in the United States and trade on U.S. exchanges ($11 \leq$ `EXCHG` ≤ 18). Also, only nonfinancial and non-public administration companies are kept (remove $6000 \leq$ `SIC` ≤ 6700 and $9000 \leq$ `SIC`). Finally, only firms with positive and non-empty entries for total assets (`ATQ`), common share equity (`CEQQ`), revenues (`SALEQ`), cost of goods sold (`COGSQ`), and SG&A (`XSGAQ`) are kept.

Profits are defined as revenues (`SALEQ`) minus cost of goods sold (`COGSQ`) and SG&A (`XSGAQ`). Aggregate profits in a given quarter is the sum of all corporate profits in that quarter. Aggregate dividends are defined as the summing profits less the first difference in gross PPE (`PPEGTQ`) across all firms in a given quarter. This is used to better match the model's definition of dividends.

Finally, I use the estimate of the one-year forward one equity risk premium from Duarte and Rosa (2015). Their estimate is an aggregation of several methods to compute the equity risk premium including the Gordon perpetual method that Farhi and Gourio (2018) utilize. I use the data from 1979 to 2020, which matches the time series of the remaining data. This time series is at the monthly level.

3 Model

This section presents a dynamic general equilibrium production-based model with capital-skill complementarities and limited participation in financial markets. This model borrows many elements from Kaltenbrunner and Lochstoer (2010), Croce (2014), and Favilukis and Lin (2016b) as well as several characteristics from standard real business cycle (RBC) models. The main modification of my model is that the production function here has capital skill-complementarities and that there are two types of households: high-skilled and low-skilled. I also restrict low-skilled households to be hand-to-mouth whereas high-skilled households have full financial market access. This setup allows me to simultaneously generate differential labor income risks and produce a stochastic discount factor that depends on high-skilled households' consumption risks only. I present the model in greater detail in the remainder of this section.

3.1 Households

The economy consists of two types of households: high-skilled and low-skilled households who correspond to college educated and non-college educated workers, respectively, in the data. Workers are exogenously endowed with skill and cannot change their skill. High-skilled households are unconstrained and have access to financial markets whereas low-skilled households are constrained and do not have access to financial markets, which implies that low-skilled households are hand-to-mouth. This assumption simplifies the model both analytically and computationally while being reasonably consistent with the data as shown before. Both types of households are representative within each type, so this economy has two representative households. All households behave competitively. I index households by skill $S \in \{H, L\}$.

This modelling approach for the households is similar to the two-agent New Keynesian model of Bilbiie (2008) with one agent who is exogenously unconstrained and the other who is constrained. I extend on this by allowing for another dimension of heterogeneity through skill. While this is a stark assumption, as shown before, high-skilled households are far more likely to participate in the stock market and hold significantly more per capita wealth in various financial instruments. Prior research such as Parker, Souleles, Johnson, and McClelland (2013), Parker (2017), and Gelman (2021) have shown that low-income and low-wealth households are significantly more likely to be hand-to-mouth. Furthermore, Eisfeldt, Falato, and Xiaolan (2022) show that college educated workers are increasingly being paid with stock or stock options.

3.1.1 High-Skilled Households

High-skilled households have Epstein and Zin (1989) and Weil (1990) preferences. They solve the standard consumption-portfolio problem given by

$$
\begin{gathered}
U_{H,t} = \max\left((1-\beta)C_{H,t}^{1-\psi^{-1}} + \beta\,\mathbb{E}_t\left[U_{H,t+1}^{1-\gamma}\right]^{\frac{1-\psi^{-1}}{1-\gamma}}\right)^{\frac{1}{1-\psi^{-1}}} \\
\text{s.t.} \\
A_{H,t+1}^{*} = (A_{H,t} + W_{H,t}N_{H,t} + D_{H,t} - C_{H,t})R_{H,t+1},
\end{gathered}
\tag{1}
$$

where γ is the risk aversion, ψ is the elasticity of intertemporal substitution (EIS), $C_{H,t}$ is their consumption at time t, $W_{H,t}$ is their wage at time t, $N_{H,t}$ is their labor supplied at time t, $A_{H,t}$ is their ex-dividend financial wealth that they enter period t with ($A_{H,t}^{*}$ is the cum-dividend financial wealth), $D_{H,t}$ is the portfolio's period t dividend payment, and $R_{H,t+1}$ is the return on the portfolio the high-skilled consumer invests in time t that is realized in $t+1$. Labor does not directly enter the utility function and is supplied exogenously and inelastically at each period t. However, across time labor supply does vary and is a function of aggregate productivity growth. This process for labor supply is identical to that of Favilukis and Lin (2016b). There are more details on the labor supply processes in Section 3.2.1.

3.1.2 Low-Skilled Households

Low-skilled households receive only labor income and are hand-to-mouth. They have the same preferences as high-skilled households. Thus their problem is given by

$$
\begin{gathered}
U_{L,t} = \max\left((1-\beta)C_{L,t}^{1-\psi^{-1}} + \beta\,\mathbb{E}_t\left[U_{L,t+1}^{1-\gamma}\right]^{\frac{1-\psi^{-1}}{1-\gamma}}\right)^{\frac{1}{1-\psi^{-1}}} \\
\text{s.t.} \\
C_{L,t} = W_{L,t}N_{L,t}.
\end{gathered}
\tag{2}
$$

The notation for the low-skilled household follows that of the high-skilled household but with the L index. Like high-skilled households, low-skilled households supply labor exogenously and inelastically at each period t that is a function of aggregate productivity growth.

3.2 Firms

There is a unit mass of atomistic competitive firms that are indexed by $j \in [0,1]$ and are all identical. Hence, there is a representative firm. However, I use the j index to denote the version of the variable that is considered a firm-level variable that firms may be able to choose. The aggregate level counterpart does not have the j index and is taken as given by firms. Firms choose investment and labor to maximize the present value of future dividend streams. Firms' stochastic discount rate is that of high-skilled households since only they hold shares of the firm.

3.2.1 Production Technology

The production function features capital-skill complementarity as in Krusell et al. (2000). Firm j's production in period t is given by

$$Y_{j,t} \equiv \bar{Z} \left(\alpha (Z_t N_{L,j,t})^{\eta\sigma} + (1-\alpha) \left[\lambda K_{j,t}^{\rho} + (1-\lambda)(Z_t \theta N_{H,j,t})^{\eta\rho} \right]^{\sigma/\rho} \right)^{1/\sigma}, \tag{3}$$

where $\bar{Z}$ is a common scaling factor for all firms, $N_{H,j,t}$ and $N_{L,j,t}$ are the labor inputs of high and low-skilled labor respectively, Z_t is the aggregate labor-embodied productivity level common to both types of labor, σ governs the elasticity of substitution between low-skilled labor and both capital and high-skilled labor, ρ governs the elasticity of substitution between high-skilled labor and capital, η is the degree of return to scale of labor, θ is a static relative efficiency factor for high-skilled labor, and α and λ are input intensity parameters that determine income shares. Capital-skill complementarities allow for a meaningful difference in labor income dynamics between high and low-skilled households relative to a standard production function with homogeneous labor. This is driven by a difference in the elasticity of substitution and input intensity.

I assume that $\rho < \sigma < 1$, that is capital is more complementary to high-skilled labor than to low-skilled labor. The elasticities of substitution are given by $1/(1-\sigma)$ and $1/(1-\rho)$. Note that this nesting structure restricts the elasticity of substitution between high-skilled workers and low-skilled workers to be the same as the elasticity of substitution between capital and low-skilled workers. This nesting structure is more consistent with estimates of the elasticities of substitution among these inputs.[6] Furthermore, the nested CES structure along with the labor-augmenting aggregate productivity factor generates a balanced growth path so that all non-stationary variables grow with Z_t.

Define $g_t \equiv \ln(Z_t/Z_{t-1})$ as the growth rate of aggregate productivity. I assume that g_t follows a stationary AR(1) process given by

$$g_{t+1} = \rho_g g_t + (1-\rho_g)\bar{g} + \varepsilon_{g,t+1}, \tag{4}$$

where $\rho_g \in [0,1)$ is the persistence parameter, $\bar{g}$ is the unconditional mean growth rate, and $\varepsilon_{g,t+1} \sim N(0,\sigma_g^2)$ is a white noise process with mean 0 and variance σ_g^2. Having a non-stationary process for the level of aggregate productivity along with Epstein and Zin (1989) and Weil (1990) preferences can endogenously generate long-run consumption and dividend risk in a production setting as shown by Kaltenbrunner and Lochstoer (2010) and Croce (2014).

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[6] See Hamermesh (1993) for more details.

Finally, recall that aggregate labor supply of both types are exactly related to aggregate productivity growth. Then, given an appropriate initial condition for $N_{H,t}$ and $N_{L,t}$, aggregate labor supply of type S can be characterized as

$$N_{S,t+1} = \rho_g N_{S,t} + (1-\rho_g)\bar{N}_S + \frac{\sigma_S}{\sigma_g}\varepsilon_{g,t+1}, \tag{5}$$

where $\bar{N}_S$ is the unconditional average labor supply of type S and σ_S is the conditional standard deviation of the labor supply of type S. The initial condition is defined such that if $g_t = \bar{g}$ then $N_{H,t} = \bar{N}_H$ and $N_{L,t} = \bar{N}_L$ for all t. Finally, define $N_t \equiv N_{H,t} + N_{L,t}$ as total employment overall.

3.2.2 Profits and Dividends

Firm j's profits in period t are given by

$$\Pi_{j,t} \equiv Y_{j,t} - W_{H,t}N_{H,j,t} - W_{L,t}N_{L,j,t} - \Xi_t, \tag{6}$$

where $\Xi_t \equiv \zeta K_t$ is a fixed operating cost that firms take as given and is scaled with aggregate capital following Favilukis and Lin (2016b) and Favilukis, Lin, and Zhao (2020). This allows the fixed cost to grow in line with the economy as this economy is non-stationary. Firms pay dividends, which are equal to profits less investment and capital adjustment costs,

$$D_{j,t} \equiv \Pi_{j,t} - I_{j,t} - \Phi(I_{j,t}, K_{j,t}), \tag{7}$$

where

$$\Phi(I_{j,t}, K_{j,t}) \equiv \frac{\nu}{2}\left(\frac{I_{j,t}}{K_{j,t}} - \delta\right)^2 K_{j,t}. \tag{8}$$

The parameter $\nu \geq 0$ governs the scale of capital adjustment costs.

3.2.3 Firm's Problem

The firm maximizes the present expected value of all future dividends and solves the following recursively defined problem

$$\begin{gathered} V_j(\Omega_t) = \max_{I_{j,t}, N_{H,j,t}, N_{L,j,t}} D_{j,t} + \mathbb{E}_t\left[M_{t+1}V_j(\Omega_{t+1})\right] \\ \text{s.t.} \\ K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t}, \end{gathered} \tag{9}$$

where M_{t+1} is the stochastic discount factor (SDF), Ω_t is a vector of state variables, and $D_{j,t}$ follows from equations (3) and (6) to (8). I go into more detail on the state variables in Section 3.3. Note that $V_j(\Omega_t)$ is also the cum-dividend value of firm j at time t with state Ω_t. I suppress the function notation of the value function from here on for ease of notation; i.e. let $V_{j,t} = V_j(\Omega_t)$. The SDF is derived from the high-skilled consumers' optimization problem. The SDF is given by

$$M_{t+1} = \beta \left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\psi^{-1}} \left(\frac{U_{H,t+1}}{\mathbb{E}_t \left[U_{H,t+1}^{1-\gamma} \right]^{(1-\gamma)^{-1}}} \right)^{\psi^{-1}-\gamma} . \tag{10}$$

After substituting the capital law of motin constraint into the objective function, the firm's first order conditions are given by

$$\begin{aligned} (K_{j,t+1}): &\quad -1 - \nu \left(\frac{K_{j,t+1}}{K_{j,t}} - 1 \right) + \mathbb{E}_t \left[M_{t+1} \frac{\partial V_{j,t+1}}{\partial K_{j,t+1}} \right] = 0, && (11) \\ (N_{H,j,t}): &\quad \frac{\partial Y_{j,t}}{\partial N_{H,j,t}} - W_{H,t} = 0, && (12) \\ (N_{L,j,t}): &\quad \frac{\partial Y_{j,t}}{\partial N_{L,j,t}} - W_{L,t} = 0. && (13) \end{aligned}$$

3.2.4 Capital Structure and Returns on Corporate Securities

The realized unlevered gross return of firm j in time $t+1$ is given by

$$R_{j,t+1} = \frac{V_{j,t+1}}{V_{j,t} - D_{j,t}}, \tag{14}$$

where $V_{j,t} - D_{j,t}$ is the ex-dividend value of firm j in t. Since this economy satisfies the conditions of Modigliani and Miller (1958), the introduction of corporate debt is relatively straightforward and capital structure does not affect the firm's value or real activity. The introduction of leverage allows the model's returns to better match its empirical counterparts.

A firm's capital structure is exogenously determined and follows the specification of Favilukis and Lin (2016b). Let $B_{j,t}$ be the market value of the firm's debt which consists entirely of one period bonds. If the firm does not default, the law of motion of firm j's debt is given by

$$B_{j,t+1} = \rho_B B_{j,t} + (1 - \rho_B) B^*_{j,t+1}, \tag{15}$$

where $\rho_B \in [0, 1)$ is a persistence parameter and $B^*_{j,t+1} \equiv \kappa(V_{j,t+1} - D_{j,t+1})$ is the target debt level for firm j in period $t+1$, and $\kappa \in (0, 1)$ is the target fraction of debt, which is constant for all firms across all time. Since all debt matures in one period, debt issued in t is paid off in $t+1$, assuming no default.

Let $R^B_{j,t+1}$ be the realized gross return on the debt $B_{j,t}$. Now $D_{j,t+1}$ is the unlevered dividend and $D^E_{j,t+1} = D_{j,t+1} + B_{j,t+1} - R^B_{j,t+1} B_{j,t}$ is the equity dividend. Thus, the realized gross equity return is given by

$$R^E_{j,t+1} = \frac{V_{j,t+1} - R^B_{j,t+1} B_{j,t}}{V_{j,t} - B_{j,t} - D_{j,t}}. \tag{16}$$

Let $R^P_{j,t}$ be the promised gross return on $B_{j,t}$ that is set in period t. If $V_{j,t+1} < R^P_{j,t} B_{j,t}$ then the firm defaults and due to limited liability equity shareholders receive a gross return of 0 in period $t+1$. Debt shareholders receive $V_{j,t+1}/B_{j,t} = R^B_{j,t+1} < R^P_{j,t}$ if the firm defaults. After the default the firm's debt is reset to 0 and the firm's debt follows (15). The firm's default does not affect production or real outcomes. If the firm does not default then $R^B_{j,t+1} = R^P_{j,t}$. Given the payout structure of debt, the promised return $R^P_{j,t}$ satisfies

$$1 = \mathbb{E}_t \left[M_{t+1} \frac{\min\{R^P_{j,t} B_{j,t}, V_{j,t+1}\}}{B_{j,t}} \right]. \tag{17}$$

3.3 Equilibrium

The vector of state variables Ω_t is given by $\Omega_t = (Z_t, g_t, K_t)$. Only the aggregate variables are needed since all firms are identical. These variables determine the present productive capacity of the economy as well as provide information on its future productive capacity.

The equilibrium consists of consumers solving programs (2) and (1), firms solving the program (9), firm beliefs, and market clearing conditions that are all mutually consistent. The aggregation conditions are given by

$$I_t = \int_{j=0}^{1} I_{j,t}\, dj, \tag{18}$$

$$K_t = \int_{j=0}^{1} K_{j,t}\, dj, \tag{19}$$

$$N_{S,t} = \int_{j=0}^{1} N_{S,j,t}\, dj \quad \forall S \in \{H, L\}, \tag{20}$$

$$Y_t = \int_{j=0}^{1} Y_{j,t}\, dj, \tag{21}$$

$$\Pi_t = \int_{j=0}^{1} \Pi_{j,t}\, dj, \tag{22}$$

$$D_t = \int_{j=0}^{1} D_{j,t}\, dj. \tag{23}$$

Conditions (18) to (23) are the investment, capital, labor supply, output, profit, and dividend aggregation conditions, respectively. Since there is a representative firm, all firm individual variables are always equal to their aggregate counterpart in equilibrium. Following these conditions, the goods market clearing conditions are given by

$$Y_t = C_t + I_t + \int_{j=0}^{1} \left[\Xi_t + \Phi(I_{j,t}, K_{j,t}) \right] dj, \tag{24}$$

$$C_{H,t} = D_t + W_{H,t} N_{H,t}, \tag{25}$$

$$C_{L,t} = W_{L,t} N_{L,t}, \tag{26}$$

where $C_t = C_{H,t} + C_{L,t}$. Given the labor supply process, labor market clearing is trivial. Finally, the risk-free asset is issued in net-zero supply and all corporate securities have one unit of supply. Since only high-skilled households participate in financial markets and there is a net-zero supply of the risk-free asset, high-skilled households always hold the market value-weighted portfolio of all corporate securities and no holdings of the risk-free asset. This implies that every period they consume both their wage bill and the aggregate dividend. Note that I assume the fixed costs and capital adjustment costs are wasted.

Policy functions for capital and labor solve the firm's problem (9) given the firm's beliefs and market clearing. The policy functions are also only functions of Ω_t. Note that the labor policy function is trivial since labor supply is inelastic. With labor markets clearing we immediately get the equilibrium labor quantity. Next, the firm has rational beliefs over high-skilled consumption $C_{H,t}$ that is a function of Ω_t. Firms also form rational beliefs over the evolution of the state variables Ω_t. Since there is no firm heterogeneity, firms know exactly the next period's endogenous state variable K_{t+1} using the current state, policy functions, beliefs, and law of motion. With this information, firms also form a rational belief over the realizations of the SDF M_{t+1}, which is a function of (Ω_t, g_{t+1}) since the realization of the next period's exogenous state variable g_{t+1} is stochastic in period t. Note that observing g_{t+1} and Z_t is sufficient to recover Z_{t+1}.

4 Calibration

The model is solved and calibrated at a quarterly frequency. Appendix C outlines how I solve the model numerically. I use three calibrations: a benchmark calibration that targets the entire time series (1979 to 2020) and two additional calibrations that target the first half (1979 to 1999) and second half (2000 to 2020) subperiods of the time series. I call these the benchmark calibration, first subperiod calibration, and second subperiod calibration, respectively. I also refer to the benchmark calibration as the full time series calibration interchangeably. Taken together this can be seen as reflecting the economy's increasing use and reliance on high-skilled workers. This approach allows me to circumvent the issues of a non-balanced growth path resulting from different growth rates in labor supply and aggregate productivity. My method is also more analytically and computationally tractable to explore asset pricing implications.

Table 1 shows the parameter values for the benchmark calibration. The calibrations for the first and second subperiods can be found in Tables A.1 and A.2 in Appendix A, respectively. Most parameters are taken from the existing literature in finance and macroeconomics but certain parameters I calibrate internally to fit targeted moments. Furthermore, the internally calibrated parameters are the only parameters that change across the different calibrations. The rest of this section is structured as follows. The remainder of this section is organized as follows. Sections 4.1 to 4.3 detail the parameter choices and Section 4.4 discusses the model fit with the targeted moments for the internally calibrated parameters. Finally, Section 4.5 describes the calibration and targeted moments for a model with homogeneous labor to compare against the baseline model.

4.1 Households

Households of both types have the same preferences. The subjective time discount factor β is set to 0.994, relative risk aversion γ is set to 7, and elasticity of intertemporal substitution (EIS) ψ is set to 2. The value of relative risk aversion is close to or consistent with that of prior studies (e.g. Jermann, 1998; Kaltenbrunner and Lochstoer, 2010; Favilukis and Lin, 2016b; Ai and Bhandari, 2021) and is within the range of values proposed by Mehra and Prescott (1985). Furthermore, since $\gamma > \psi^{-1}$, this implies that households have a strict preference for an early resolution of risk and is also within the usual range in the literature. Bansal and Yaron (2004) show that an EIS greater than one is needed for the long-run risk channel to be consistent with various asset pricing facts. This combination of γ and ψ allows this class of models to generate a reasonable risk premium and Sharpe ratio. Moreover, along with β these parameters result in a relatively low and stable risk-free rate.

4.2 Firms

The elasticities of substitution are $1/(1-\sigma) \approx 1.67$ and $1/(1-\rho) \approx 0.67$, which follow the estimates of Krusell et al. (2000). These estimates (or similar values) have been utilized in various prior studies featuring capital-skill complementarities (e.g. Caselli and Coleman II, 2002; Lindquist, 2004; Ben-Gad, 2008; Dolado, Motyovszki, and Pappa, 2021, among others). Furthermore, these estimates are fairly robust. Polgreen and Silos (2008), Ohanian, Orak, and Shen (2021), and Maliar, Maliar, and Tsener (2022) update the data series and all find similar estimates for the elasticities of substitution. Only Castex, Cho, and Dechter (2022) finds a moderate increase in ρ in recent years, implying a moderate decline in the complementary relation between capital and high-skilled labor. I discuss the implications of this in Section 5.3; for now I fix σ and ρ across all calibrations.

The parameters α and λ jointly reflect the intensity of each input in the production technology and with η these directly determine income shares. In a Cobb-Douglas setting in which $\sigma \to 0$ and $\rho \to 0$, the income shares for high and low-skilled labor are given by $(1-\alpha)(1-\lambda)\eta$ and $\alpha\eta$, respectively. However, in a generalized CES setting, the income shares are not characterized by such a straightforward analytical expression. Furthermore, along with σ and ρ these parameters determine the skilled wage premium. Since wages are equal to the marginal product of labor in the model, the skilled wage premium at time t is given by

$$\frac{W_{H,t}}{W_{L,t}} - 1 = \frac{(1-\alpha)(1-\lambda)}{\alpha} \left(\lambda K_t^{\rho} + (1-\lambda)\left(Z_t \theta N_{H,t}\right)^{\eta\rho}\right)^{\sigma/\rho-1} \frac{(Z_t\theta)^{\eta\rho} N_{H,t}^{\eta\rho-1}}{Z_t^{\eta\sigma} N_{L,t}^{\eta\sigma-1}} - 1. \tag{27}$$

There is not a standard calibration for α, λ, and η so these parameters are internally calibrated, which is discussed in more detail later. The final parameter in the firm's production technology is $\bar{Z}$ which sets the level and is set to 0.25. This parameter value is set to be the mean value of the idiosyncratic productivity process of Favilukis and Lin (2016b) and Favilukis, Lin, and Zhao (2020) for comparability.

The depreciation rate of capital δ is set to 0.025, which implies a quarterly depreciation rate of 2.5% and an annual depreciation rate of approximately 9.6%, which is within the standard values in macroeconomics. Along with α and λ, δ also determines the investment-to-output ratio. I internally calibrate α, λ, and η by targeting the low-skilled labor share, high-skilled labor share, skilled wage premium, and investment-to-output ratio.

The firm faces quadratic capital adjustment costs and a fixed operating cost. Increasing ν and ζ increases both the equity risk premium and equity return volatility by limiting the countercyclical effect of investment and increasing operating leverage, respectively. Jermann (1998) shows the importance of capital adjustment costs to simultaneously match investment volatility and the equity risk premium. Capital adjustment costs limit the firm's ability to utilize investment and future capital to generate countercyclical dividends. Capital adjustment costs are scaled by ν, which is internally calibrated by targeting the volatility of investment growth relative to the volatility of output growth. The fixed operating cost follows the same specification as Favilukis and Lin (2016b). The scale factor ζ is internally calibrated to target the average unlevered market-to-book ratio of 1.33, which follows the approach of Zhang (2005), Favilukis and Lin (2016b), and Favilukis, Lin, and Zhao (2020). Across the calibrations the proportion of fixed costs relative to total output ranges from 21.2% to 21.4%, which is consistent with the values estimated by De Loecker, Eeckhout, and Unger (2020).

Finally, the remaining parameters related to the firm are for its capital structure. The process for the one period corporate debt follows the calibration of Favilukis and Lin (2016b). Favilukis and Lin (2016b) estimate that the average leverage ratio using Compustat firm is 0.37 and set the model's target leverage ratio κ to this. The persistence of debt $\rho_B = 0.90$ is also estimated using Compustat data.

4.3 Aggregate Productivity Growth and Labor Processes

Aggregate productivity has an average growth rage $\bar{g} = 0.005$, which is a standard value in the literature. I use the same persistence parameter $\rho_g = 0.8$ as Favilukis and Lin (2016b). I set the conditional standard deviation σ_g to 0.01 to match the standard deviation of annual private GDP growth from 1945 to 2021, which is approximately 4.49%. The output growth volatility in the benchmark calibration is 4.61% (4.66% and 4.50% in the first and second alternative calibrations, respectively). Recall that aggregate labor supply of both types can be modeled as AR(1) processes with the same persistence and error process as aggregate productivity growth but with different unconditional means and conditional standard deviations.

The unconditional means $\bar{N}_H$ and $\bar{N}_L$ are chosen so that they sum to one as a normalization so that these parameters can be interpreted the proportion of the labor supply that is high-skilled and low-skilled, respectively. As a result, $\bar{N}_S$ is chosen to match the proportion of type S employment in the data. The conditional standard deviations are chosen to match the relative employment growth volatility of type S to aggregate output growth volatility. I express the conditional standard deviations of the two labor supplies as

$$\sigma_H = \bar{N}_H \sigma_N, \tag{28}$$

$$\sigma_L = \bar{N}_L \Sigma_L \sigma_N, \tag{29}$$

where σ_N is the common standard deviation scaling factor for the labor supplies and Σ_L is a parameter governing the relative conditional standard deviation of low-skilled labor supply growth to that of high-skilled labor. I scale the conditional standard deviations by the unconditional means; thus, if $\Sigma_L = 1$ then both labor supplies have the same growth volatility. This formulation also demonstrates how I conduct a comparative statics exercise later in Section 5.3. In the calibration, low-skilled households have more variable employment than high-skilled households, that is $\Sigma_L > 1$. This is consistent with the findings of Bils (1985) and Doniger (2021).

4.4 Internally Calibrated Parameters and Targeted Moments

The vector of internally calibrated parameters is $(\alpha, \lambda, \eta, \nu, \zeta, \bar{N}_H, \bar{N}_L, \sigma_N, \Sigma_L)$.[7] These nine parameters are chosen to fit ten targeted moments. Table 2 shows the targeted moments and model fit for the baseline model and its various calibrations.[8] The baseline model across all calibrations is able to fit the targeted moments well. Across calibrations only a subset of the internally calibrated parameters change; all other parameters remain constant.

I make several adjustments to the standard macroeconomic data series to generate a better comparison with the model output. As previously mentioned, my model does not feature a government sector. Thus, I use private GDP from the data, which is defined as GDP less government expenditures, as the empirical counterpart. Private GDP growth volatility is the denominator for all relative growth volatility figures in Table 2. I utilize a similar analog for the wage bill

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[7] Equivalently, I can pick σ_H and σ_L directly instead of σ_N and Σ_L. Given $\bar{N}_H$ and $\bar{N}_L$, choosing σ_H and σ_L implies values for σ_N and Σ_L and vice versa following equations (28) and (29).

[8] I show the overall labor share in Table 2 for convenience but including this does not impose an additional restriction on the calibration as it is a perfect linear combination of the labor shares by skill both in the data and in the model.

and employment statistics; that is I use only the compensation for those employed in the private sector and private sector employment figures. Next, in my model wages are similar to the total labor compensation for a given period, thus in the data I define wages as the wage bill divided by employment in that period. Finally, in my model investment is most similar to nonresidential investment in the data, thus the investment relative volatility and investment-to-output ratio are computed using this figure.

Note that throughout the calibrations, I target the same relative growth volatilities (investment, high-skilled employment, and low-skilled employment) and market-to-book whereas the remaining moments are specific to each calibration. I target the same relative growth volatilities for these variables since there is no clear trend across the time series for these figures. Similarly, as shown by Farhi and Gourio (2018), valuation ratios have been relatively stable in the time series as well, thus I target the same market-to-book ratio across calibrations. The remaining moments, however, have exhibited clear trends across the time series.

4.5 Calibration of Model with Homogeneous Labor

I also solve a model that does not feature capital-skilled complementarities and hence one type of skill for comparison. In this setting households only differ in their participation in financial markets, which is exogenous. This model retains all other features in the previous model. I call this model the model with homogeneous labor and simple model interchangeably. The simple model utilizes analogous notation for ease of exposition. Appendix D.1 presents the simple model in greater detail.

Similar to the baseline model, I employ three calibrations with a similar strategy. Since the simple model cannot reflect changes in the skill composition in the labor force, the only parameter that varies across calibrations is a parameter reflecting the proportion of the population that participates in financial markets. Participation in financial markets is exogenous and fixed. The proportion of the population participating is denoted by $\tau \in (0, 1]$. The benchmark calibration features $\tau = 1$ (full participation). The first subperiod calibration sets $\tau = 0.2$ and the second subperiod calibration sets $\tau = 0.3$, to reflect the change in the proportion of the total population participating in the stock market as shown in Figure 2.

Table 3 presents the parameters of the simple model that are different from those of the baseline model. The only parameters that are different are related to the firm's production technology, fixed operating costs, capital adjustment costs, and labor supply. Within the subset of parameters shown in Table 3 only σ is calibrated externally and the labor supply is normalized to have an unconditional mean of 1. The remainder are calibrated internally by targeting certain moments. The elasticity of substitution between capital and labor is given by $1/(1 - \sigma) \approx 0.90$, which implies that in the simple model's setting labor and capital are gross complements.

There is significant variation in the estimates of the elasticity of substitution as documented by León-Ledesma, McAdam, and Willman (2010), though most estimates are below one and can soundly reject a Cobb-Douglas production function. In fact many earlier studies tend to find the elasticity of substitution to be less than 0.6. Using these values, however, would imply that homogeneous labor is more complementary to capital than even high-skilled labor with capital-skill complementary, which is paradoxical. León-Ledesma, McAdam, and Willman (2010) show that part of the variation in the estimates can be explained by the various estimation techniques used. Through Monte Carlo simulations they show that utilizing a system of multiple equations yields the closest estimates.

Given these findings, I draw from some recent studies estimating the elasticity of substitution to choose σ. Herrendorf, Herrington, and Valentinyi (2015) estimate the elasticity of substitution for the aggregate U.S. economy as well as various sectors using a system of multiple equations. They estimate the elasticity for the aggregate U.S. economy to be 0.84. Karabarbounis and Neiman (2014) estimate the elasticity using a wide cross-section of countries to explain the decline of the labor share and estimate that the elasticity of substitution is 1.25, which implies that a decrease in the price of capital results in a decrease in the labor share. Glover and Short (2020) update the dataset of Karabarbounis and Neiman (2014) by considering more countries and including consumption growth in the regression specification. They show that the omitted variable bias leads the elasticity of substitution to be overestimated. Their average point estimate is 0.973, slightly below one. The value $\sigma = -0.111$ with an elasticity of substitution of 0.9 is within the range of recent estimates and also is in between the elasticities of substitution for the model with capital-skill complementarities.

The parameters $(\alpha, \eta, \nu, \zeta, \sigma_N)$ are internally calibrated; the unconditional mean labor supply is normalized to one and thus does not have an explicit targeted moment. A total of five moments are used. Table 4 shows the model fit for these targeted moments; the overall model fit is good.[9] For all calibrations of the simple model, the annual output growth volatility is 4.40%.

5 Results

In this section I examine the model's implications and results across the calibrations. I also compare this model's results with the results of the model with homogeneous labor. I start by comparing the two models' performance on a wide array of macroeconomic and financial moments. This provides an external check on the models beyond the targeted moments. Furthermore, I can check whether my calibration strategy of targeting various trends, such as the labor share and composition of skills, can also result in matching the trends in the equity risk premium and risk-free rate. This exercise also provides the initial insights into the key mechanisms driving the results. Finally, I decompose the results by conducting a comparative statics exercise on key parameters to further inspect the economic mechanism.

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[9] Here the overall labor share is a meaningful restriction as the model with homogeneous labor has no notion of differentiated skill for labor.

5.1 Model Comparison

5.1.1 Macroeconomic Moments

I start this section by discussing the macroeconomic moments produced by the baseline mode and comparing it to the macroeconomic moments generated by the simple model. Table 5 shows the benchmark calibration's macroeconomic moments. Tables A.3 and A.4 show these moments for the first subperiod and second subperiod calibrations, respectively. Note that all moments shown are at the annual level. Overall the baseline model across calibrations is able to match most macroeconomic moments. The simple model across calibrations (Tables A.5 to A.7) is also able to match many standard macroeconomic moments though with some shortcomings relative to the baseline model.

As with many macroeconomic models with frictionless wages, wages are too volatile relative to what is observed in the data.[10] This is in part driven by the fact that wages are always equal to the marginal product of labor, which is volatile.[11] Both models also produce moments for the wage bills that match what is found in the data but also produce insufficiently volatile but countercyclical labor shares.

The baseline model, however, can produce labor moments by skill whereas the simple model cannot, by construction. The moments for high-skilled labor (wage, wage bill, labor share) match the data much more closely than the low-skilled counterparts. While there are some issues with the performance of various labor moments as is standard in many RBC models, since consumers are representative by type, the fact that the wage bill dynamics are close is the most important. The wage bill here is closer to a household's total labor income in a representative household setting with inelastic and exogenous labor supply.

The simple model's wage bill dynamics does not change across calibrations. The overall wage bill volatility is 4.32% across calibrations. However, the baseline model is able to generate wage bill that is declining in volatility (from 4.52% to 4.28%.). This implies that aggregate labor income risk has decreased, which is in line with prior findings (e.g. Braxton et al., 2021). However, the baseline model indicates that high-skilled labor income risk has increased, which is also consistent with prior empirical evidence. The high-skilled wage bill volatility increases from 3.40% to 3.56%. By construction the simple model cannot replicate these dynamics.

Another key shortcoming among the macroeconomic moments of the simple model and its calibrations relative to the baseline model and its calibrations is that consumption is too volatile. While both models generate this outcome, there are important differences in how this result is produced. In the baseline model only the consumption risk of high-skilled households directly matter for asset pricing implications. The benchmark calibration produces a consumption volatility of

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[10] See Shimer (2005), Hall (2005b), Hall (2005a), and Gertler and Trigari (2009) among others for a more detailed discussion of solutions for this.

[11] Note that in the baseline model wages overall are defined as $W_t = \left(W_{H,t}N_{H,t} + W_{L,t}N_{L,t}\right)/N_t$. This matches the procedure in the data.

3.99% or a relative volatility of 0.866 to private GDP. The first and second subperiod calibration generate high-skilled consumption volatility of 3.79% and 4.15%, respectively.[12] Conversely, in the simple model across all calibrations (all levels of participation) the consumption volatility of stockholders is roughly equal to that of output (4.35%). Generating a consumption volatility that is higher than the aggregate consumption in the data is desirable since it is true that stockholders' consumption is higher than that of nonstockholders (e.g. Mankiw and Zeldes, 1991; Malloy, Moskowitz, and Vissing-Jørgensen, 2009). However, with full participation ($\tau = 1$) this result is produced, indicating that the simple model generates a higher risk premium and lower risk-free rate counterfactually through excessive consumption volatility. Note that both models do counterfactually produce a higher nonstockholder consumption volatility due to the hand-to-mouth assumption. Since the main focus is on asset pricing implications, however, this outcome while not desirable is not key for the main findings and intuition.

5.1.2 Profit and Dividend Moments

Standard RBC models struggle to produce sufficiently volatile and procyclical dividends. This is in part due to profits not being sufficiently volatile due to the wage bill acting as a countercyclical force. This transmits to smoothing dividends. Furthermore, investment acts as a countercyclical weight that smooths dividends. In fact under certain calibrations generate countercyclical dividends. Table 6 shows the profit and dividend moments from both the data and models. In the data profits are taken from the universe of U.S. Compustat firms from 1979 to 2020. Recall that firm-level profits are defined as sales minus COGs and SG&A and are then summed to represent the economy's total corporate profits. Aggregate profits have a volatility of 10.3% in the data across the whole time series. In the data, growth volatility of profits increases from 10.0% in the first subperiod to 10.9% in the second subperiod. Both models generate insufficiently volatile profits that are too procyclical as measured by the correlation with output growth. The baseline model generates a profit volatility of 8.8% in the full time series (8.7% in the first subperiod and 9.0% in the second subperiod). The simple model generates a volatility of approximately 8.3% across all calibrations. The baseline model is able to better replicate the changes in profit volatility across subperiods.

The changes in profit volatility across calibrations for the baseline model are driven primarily by changes in the wage bill volatility. Since the wage bill is procyclical and a cost, it acts as a smoothing onto profits and dividends. The wage bill is smoother in the second subperiod calibration relative to the first subperiod calibration. This reduces the countercyclical pressure it exerts and results in more volatile profits. This relationship is shown in Figures 6 and 7.[13] These figures plot the impulse response function of key variables with respect to a positive aggregate productivity growth shock

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[12] The relative volatilities are 0.813 and 0.922, respectively.

[13] See Figures A.1 and A.2 in the Appendix for the impulse response functions for all three calibrations for the same set of variables. Figures A.1 and A.2, correspond to Figures 6 and 7, respectively.

that lasts for one quarter. As shown in Figure 6, output and consumption responds similarly across calibrations for both models. However, the wage bill is somewhat less responsive in the second subperiod calibration than in the first subperiod calibration for the baseline model (Panel (a) of Figure 7). For the simple model, there is almost no difference in sensitivity (Panel (b) of Figure 7). This tranmits to more volatile profits (Panel (c) of Figure 7). This effect is significantly more pronounced for dividends (Panel (e) of Figure 7) which also includes the effect of investment.

Dividends are defined as profits less the first difference in gross PPE in the data to match the model's definition. Dividends become more volatile in the data from the first subperiod to the second subperiod. They increase from 28.9% to 39.7% with a volatility of 34.4% across the whole time series. Both models also generate insufficiently volatile dividends that are too procyclical. Furthermore, both models counterfactually generate dividends that are less volatile than profits. However, this issue is far more pronounced with the simple model than for the baseline model. The baseline model generates a dividend volatility ranging from 5.4% to 7.6% whereas the simple model generates a dividend volatility ranging 4.7% to 4.8%; the baseline model's change in dividend volatility matches the change in the data more closely. Part of this can be attributed to the fact that the baseline model tends to produce a slightly more volatile profit. However, most of this difference is attributable to investment. In the baseline model, investment exerts a smaller countercyclical force despite matching the same volatility moments. This becomes more pronounced as the economy depends more on high-skilled labor.

5.1.3 Financial Moments

Finally, I discuss the financial moments that both models generate. Table 7 shows the financial moments for the data and the models across all subperiods and calibrations. I also include the financial moments from an exchange model. Appendix D.2 provides more details on the exchange model. Both the baseline and simple models can generate low and stable risk-free rates and sizeable Sharpe ratios due to the endogenous long-run risk mechanism. The one-year constant maturity U.S. Treasury bill yield is used as the proxy for the risk-free rate in Column (1). Column (6) contains the one-year ahead expected equity premium using data from Duarte and Rosa (2015); the model also produces a similar counterpart that is also shown in Column (6). The sum of the risk-free rate and the one-year ahead expected equity risk premium yields the expected one-year ahead equity return in Column (3). The volatility of equity returns and the realized premium equity (column (4) and (7), respectively), however, is estimated using the realized total market returns from Kenneth French's website. Similarly, I use the realized equity returns and equity premium produced from the model simulation to compute the model counterparts.

The baseline model generates a higher risk-free rate since the consumption of stockholders (high-skilled households) is less volatile than that of the simple model (1.558% and 1.431%, respectively, in the benchmark calibrations). However, the baseline model is able to generate a sizeable equity risk premium; in some calibrations the baseline model produces a slightly higher equity risk premium than the simple model. All calibrations for both the baseline model and simple model generate an equity premium above 4%. Though the equity risk premium generated by the models, like many other general equilibrium production models, is lower than what is observed in the data. Similarly, the volatility of equity returns and realized equity premium are also too low.

However, the baseline model is able to generate a dividend term structure that is more consistent with the empirical evidence (e.g. van Binsbergen, Brandt, and Koijen, 2012; van Binsbergen, Hueskes, Koijen, and Vrugt, 2013; van Binsbergen and Koijen, 2017) than what the simple model produces. Figure 8 shows the expected returns and return volatility of dividend strips up to 200 quarters in the future for both models across calibrations.[14] The dividend term structure of the baseline across all calibrations initially slopes upwards but within ten periods the dividend term structure is downwards sloping. This is true for both the expected returns and return volatility. For the benchmark calibration, short-run dividends earn an quarterly expected return that is approximately 0.20 percentage points (p.p.) higher that the quarterly expected return of long-run dividends as shown in Panel (a) of Figure 8. This difference is more pronounced in the second subperiod calibration and less pronounced in the first subperiod calibration. The simple model, however, produces a far less pronounced downwards slope after approximately ten quarters across all calibrations for both expected returns and return volatility (Panels (b) and (d), respectively in Figure 8).

Long-run risk models generally produce upwards sloping dividend term structures, as van Binsbergen, Brandt, and Koijen (2012) show. Since most calibrations imply that most of the risk is through long-run consumption growth risks. Given that consumption and dividends are cointegrated, long-run dividends are highly loaded on this risk. In standard long-run risk models, short-run risks are not highly correlated with long-run risks and results in less risky short-run dividends.

There are two factors that result in the baseline model to produce overall downwards sloping dividend term structure. The first is that the productivity process in the model is simpler than that of Bansal and Yaron (2004). The standard productivity process features explicitly two sources of risk: a short-run risk and a long-run risk. Similar to Favilukis and Lin (2016b), the productivity process in my model only features one source of risk that effectively implies that the short-run and long-run risk are perfectly correlated. This also generates riskier short-run dividends. However, this alone is not enough to explain the term structure and both the baseline and simple model feature the same productivity process.

The second and more important reason is related to the procyclicality and volatility of the dividend processes across models and calibrations. While both models are able to generate procyclical dividends, the baseline model generates more volatile dividends and from the first

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[14] See Appendix C.2.5 for more details on the computation of the expected returns and return volatility of the dividend strips.

subperiod calibration to the second subperiod calibration generates more volatile dividends. The increase in capital-skill complementarity and reliance on high-skilled labor results in a more volatile dividend. A more volatile dividend process increases the riskiness of short-run dividends. Since the determinants of dividends such as output, wages, and investment are all cointegrated, dividends are expected to return to its normal share of output in the long-run since productivity growth is mean reverting.[15] Therefore, this can result in long-run dividends being less risky than short-run dividends and results in a downwards sloping dividend term structure.

Finally, the baseline model is able to replicate the recent trends in the equity premium and risk-free rate whereas the simple model cannot. From the first subperiod (Panel B of Table 7) to the full time series (Panel A of Table 7) to the second subperiod (Panel C of Table 7), the baseline model produces an increasing equity risk premium and decreasing risk-free rate. The equity premium increases by 0.540 p.p. in the baseline model from the first subperiod to the second subperiod, which is approximately 19.4% of the increase in the data. Furthermore, the risk-free rate decreases by 0.129 p.p. across the same calibrations, but this only represents 3.9% of the change in the data. These results are in contrast to those of the simple model, which produces a slightly increasing risk-free rate and essentially stable equity risk premium as τ increases. The change across calibrations for the simple model is not dramatic relative to the results of Basak and Cuoco (1998) or Constantinides and Duffie (1996) because in this model participation is for all assets and there is nontrivial production to generate the dividend process. Thus, the model lacks a leverage effect that amplifies the effect and has a general equilibrium response that further dampens and even offsets the direct effect of participation. The exchange model demonstrates the pure impact of changing the degree of financial market participation without the general equilibrium response in dividends. As participation increases the risk premium decreases and risk-free rate increases, consistent with prior results and intuition. Increasing participation from $\tau = 0.2$ to $\tau = 0.3$ decreases the equity risk premium by 0.064 p.p. and increases the risk-free rate by 0.063 p.p.; an increase from $\tau = 0.2$ to $\tau = 1$ leads to a 0.179 p.p decrease in the equity risk premium and 0.176 p.p increase in the risk-free rate.

5.2 Economic Mechanism

Given the two models' performances in generating or matching various moments, I further discuss the underlying mechanisms that allow the baseline model to better replicate these facts. First I discuss how the changing production technology and inputs affect the volatility of profits and dividends which in turn transmits to asset prices. I also analyze how matching various other macroeconomic trends such as the declining labor share and lackluster investment contribute to

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[15] Belo, Collin-Dufresne, and Goldstein (2015) and Favilukis and Lin (2016b) feature a similar cointegration mechanism to generate a downwards sloping dividend term structure. Belo, Collin-Dufresne, and Goldstein (2015) feature a mean-reverting financial leverage policy, which generates a downwards slope for equity dividends. Favilukis and Lin (2016b) show that wage rigidity can produce less procyclical wages and wage bill, which in turn generates a more procyclical dividend and downwards sloping dividend term structure for both unlevered and levered dividends.

the trends in the equity risk premium and risk-free rate in the context of this model. Finally, I discuss the general equilibrium response in this model and how that contributes to the overall results.

As the baseline model becomes more dependent on high-skilled labor overall output becomes smoother (4.66% in the first subperiod to 4.50% to the second subperiod). However, profits become more volatile in the same transition because the wage bill becomes sufficiently smoother and relatively smaller. The overall wage bill's volatility decreases from 4.52% in the first subperiod to 4.28% in the second subperiod. Furthermore, the overall wage bill is relatively smaller as the labor share declines from 59.32% to 57.97% (a 2.28% relative decline), matching the trend in the data. Since the wage bill is still procyclical, therefore still smoothing profits since it is a cost, a smaller labor share further reduces this smoothing effect onto profits. The wage bill becomes smoother as a greater share of the cost is attributable to high-skilled labor. The high-skilled wage bill is smoother than the low-skilled wage bill due to the complementary relation with capital.[16] Thus, as more of the wage bill is paid to high-skilled workers, the wage bill becomes smoother. One issue with this result is that in the data the high-skilled wage bill is only slightly smoother than the low-skilled wage bill, however, it is significantly less procyclical. However, shifting the composition to make the wage bill less procyclical would have a similar effect.

The increase in profit volatility transmits to an increase in dividend volatility. Dividend volatility is also determined by investment volatility. The effect of the change in investment on dividends is similar to the effect of the change in the wage bill on profits. This result from the model links the recent trend of lackluster investment to increased risk premia and decreased risk-free rates. While the change in investment volatility and the investment-to-output ratio is somewhat small in the model and in the data, since investment is relatively large compared to dividends, even small changes can result in a large effect.

The model's results are also driven by a general equilibrium response. In equilibrium the high-skilled household's consumption is the sum of the wage bill to high-skilled labor and the aggregate dividend. Since only high-skilled household's consumption risk determines the SDF, both the risk dynamics of the high-skilled wage bill and dividend implicitly matter for asset prices through how they affect consumption risk. Similar to many general equilibrium asset pricing models, even those with trivial production sectors, having a more volatile dividend not only raises risk premia on the production or asset side but also through the consumption side by making consumption riskier as dividends are a component of consumption.

However, in this model as the economy grows more dependent on high-skilled labor, the labor income risk of high-skilled workers increases, as previously discussed. This is represented by an increase in the volatility of the high-skilled wage bill, which is the total labor income of high-skilled households. From the first subperiod calibration to the second subperiod calibration the volatility

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[16] Favilukis and Lin (2016b) show how the volatility of wages change in standard CES production technology setting as the elasticity of substitution changes.

of the high-skilled wage bill increases from 3.40% to 3.56%, an increase of 4.65%. Figure 9 also reflects this difference in the impulse response functions of the wage bills by skill.[17] In Panel (a) of Figure 9, the high-skilled wage bill is more sensitive to the productivity shock in the second subperiod calibration whereas the low-skilled wage bill's sensitivty is relatively similar across calibrations as shown in Panel (b) of Figure 9. This result is consistent in a reduced form manner to the findings of Braxton et al. (2021) and Kogan et al. (2022). This is driven by the increase in capital-skill complementarities. Despite the high-skilled wage bill becoming more volatile, the shift towards high-skilled labor still smooths the overall wage bill since the high-skilled wage bill is less volatile than the low-skilled wage bill. This implies that the compositional effects offset the within type increase in volatility. The combined effect of the high-skilled wage bill and dividend leads to high-skilled consumption becoming more volatile (an increase from 3.79% to 4.15%, a 9.53% increase). A more volatile high-skilled consumption leads to a more volatile SDF. This leads to a lower risk-free rate since $R_{f,t+1} = \mathbb{E}_t[M_{t+1}]^{-1}$. Furthermore, this results in a higher risk premium.

In contrast, the simple model fails to generate this trend because the only change is the proportion of the population that participates in financial markets τ. Changing τ does not have any significant impact on labor income risk and has a small impact on dividends. However, changing τ changes risk sharing by affecting the proportion of consumption of stockholders that is derived from labor income and dividend income. Increasing τ places a higher weight on labor income, which is less risky and volatile than dividends, which leads to a less volatile SDF. This decreases the risk premium and increases the risk-free rate. The change in risk sharing also has some changes on the production side of the economy. The increase in τ means that stockholders are willing to bear greater dividend risk. In equilibrium the firm generates somewhat more volatile dividends as part of the value maximization problem. The effect of the slightly riskier dividend is countered by the greater risk sharing leading to an ambiguous effect on the risk premium and risk-free rate. The exchange model more clearly shows the effect of increased risk sharing resulting from higher participation without also introducing the effects of changing the SDF onto the firm's problem.

5.3 Comparative Statics

Several parameters change across calibrations to match various targeted moments. I conduct a comparative statics exercise for key parameters to decompose the economic mechanism. Throughout the comparative statics exercise only one parameter is changed with all other parameters being fixed at the values specified in the benchmark calibration in Table 1. The most important changes are the changes to the production technology's input intensity (α and λ) and the increase in the proportion of high-skilled labor ($\bar{N}_H$). The parameters α and λ govern the input intensity of low-skilled labor and capital within the inner nest, respectively, whereas $\bar{N}_H$ not only determines the proportion of high-skilled labor but also the degree of financial market participation.

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[17] See Figure A.3 in the Appendix for the version of Figure 9 showing all calibrations.

5.3.1 Effect of Changing Input Intensity

The parameters α and λ decrease monotonically going from the first subperiod calibration to the second subperiod calibration. Decreasing α decreases the input intensity of low-skilled labor and increases the input intensity of both capital and high-skilled labor. Whereas decreasing λ only decreases the input intensity of capital and increases the input intensity of high-skilled labor with no effect on the input intensity of low-skilled labor. Therefore, the joint movement reflects the decreasing input intensity of low-skilled labor and increasing input intensity of high-skilled labor. The joint effect of these two parameters is generally ambiguous on the input intensity of capital.

First, I examine the comparative statics of α and λ on the volatility of profit growth and its primary determinants (output, the overall wage bill, and labor share). Panel (b) of Figures 10 and 11 show that profit growth volatility is decreasing α and increasing in λ, respectively. As α decreases and λ increases, the input intensity of capital increases. Since capital is the only nontrivially chosen input in firm production, more capital intensity implies that firms can better utilize capital to generate smoother output following the value maximization problem. However, the firm cannot generate smoother profits (and then in turn smoother dividends), due to the changing capital-skill complementarities effect on its inputs and costs.

Profits become less/more volatile despite output becoming more/less volatile (Panel (a) of Figures 10 and 11) for α and λ, respectively. This implies that the countercyclical pressure of the overall wage bill becomes sufficiently stronger/weaker to offset the effect of output volatility for α and λ, respectively. Panel (c) of Figures 10 and 11 show that increasing α increases the volatility of the wage bill and increasing λ decreases the volatility of the wage bill. Furthermore, the change in the labor share also contributes to offsetting the effect of output volatility on profit volatility. The labor share is increasing in α and decreasing in λ (Panel (d) of Figures 10 and 11, respectively). As previously discussed, α and λ are key determinants of income shares and this comparative statics result is consistent with usual intuition. The combined effect of a more volatile and relatively larger wage bill offsets the increase in volatility of output to generate a smoother profit as α increases. The opposite is true for λ.

Figures 12 and 13 decompose the comparative statics of α and λ, respectively, on the components of the wage bill and labor share by skill. Increasing α has a small increasing effect on the volatility of the high-skilled wage bill and has a decreasing effect on the high-skilled labor share (Panels (a) and (b) of Figure 12). While increasing λ also decreases the high-skilled labor share, the high-skilled wage bill becomes smoother, which is shown in Panels (a) and (b) of Figure 13. The effect of λ is quantitatively larger than that of α so jointly decreasing both increases the volatility of the high-skilled wage bill while increasing the high-skilled labor share. The low-skilled wage bill becomes more volatile with respect to α and less volatile with respect to λ as seen in Panel (c) of Figure 12. Increasing α and λ has the same qualitative effect on both wage bills by skill type, which leads to the increasing/decreasing volatility of the overall wage bill, respectively.

Finally, the volatility of dividends is primarily determined by the volatility of profits and investment. Figures 14 and 15 show the comparative statics of α and λ on dividends, investment, and various key financial moments. In Panel (a) of both Figures 14 and 15 show that dividend volatility is decreasing in α and increasing in λ. This is driven by investment becoming more volatile with respect to α and less volatile with respect to λ, respectively (Panel (b) of Figures 14 and 15). The investment-to-output ratio (Panel (c) of Figures 14 and 15) is decreasing in α and increasing in λ, this partially offsets the effect of change in investment volatility for both parameters and drives the curvature of the dividend comparative statics. As α increases and λ decreases, the intensity of capital as an input decreases. This decreases the use of capital/investment as seen in the lower investment-to-output ratio. However, the growth of investment becomes more volatile as the optimal investment needs to respond relatively more to other inputs whose input intensity has increased.

The combination of dividend and high-skilled wage bill volatility feeds directly into the volatility of high-skilled consumption. Given that $R_{f,t+1} = \mathbb{E}_t[M_{t+1}]^{-1}$, there is a one-to-one inverse relationship with the risk-free rate and high-skilled consumption growth volatility. High-skilled consumption growth volatility is increasing in α as the decrease in dividend volatility offsets the increase in the high-skilled wage bill volatility. For λ, high-skilled consumption growth volatility is decreasing as the decrease in the high-skilled wage bill volatility offsets the increase in dividend volatility. This leads to the risk-free rate being decreasing in α and increasing in λ (Panel (d) of Figures 14 and 15). The expected equity return is decreasing in α and λ (Panel (e) of Figures 14 and 15). Since for both parameters the direction of dividend volatility is opposite that of the SDF volatility, this implies that the effect of the change in dividend volatility offsets the effect of the change in SDF volatility. Similarly, the change in the expected equity return is generally large enough to offset the opposing change in the risk-free rate so that the expected equity risk premium has qualitatively similar comparative statics (Panel (f) of Figures 14 and 15).

Given the curvature of the expected equity return and expected equity premium comparative statics with respect to α and λ, the effect of α offsets the effect of λ when both are decreasing. Therefore, the input intensity shift in the production technology from low-skilled labor to high-skilled labor contributes to a higher equity risk premium. Similarly, the effect of λ offsets the effect of α to make the high-skilled wage bill more volatile. The two parameters, however, seem to approximately offset each other for the risk-free rate. Overall, in the context of this model α and λ contribute significantly to replicating the trends in the equity risk premium while still matching the key macroeconomic trends such as the declining labor share and lackluster investment.

5.3.2 Effect of Changing Proportion of High-Skilled Labor

The comparative statics exercise with $\bar{N}_H$ is performed slightly differently than that for the other parameters. To preserve the interpretation of $\bar{N}_H$ and $\bar{N}_L$ being the unconditional mean proportions of labor, I always maintain that $\bar{N}_H + \bar{N}_L = 1$. Thus, an increase in one implies a decrease in the other. This comparative statics exercise reflects the changing quantities of the two labor inputs that firms use in production. Furthermore, increasing $\bar{N}_H$ also increases the level of financial market participation in the economy as the model assumes only high-skilled households participate.

The increase in $\bar{N}_H$ reduces the overall volatility of labor employment since high-skilled labor supply is less volatile than low-skilled labor supply. This leads to output becoming smoother as $\bar{N}_H$ increases (Panel (a) of Figure 16). Similar, to the comparative statics of α and λ, profit volatility changes in the opposite direction of output volatility with respect to $\bar{N}_H$ as shown in Panel (b) of Figure 16. The wage bill becomes smoother (Panel (c) of Figure 16) and labor share becomes smaller (Panel (d) of Figure 16), which lead to a more volatile profit.

Figure 17 decomposes the effect of $\bar{N}_H$ on the wage bill and labor share by skill. In Panels (a) and (c) of Figure 17 both the high-skilled wage bill and low-skilled wage become smoother as $\bar{N}_H$ increases. However, the high-skilled labor share is increasing whereas the low-skilled labor share is decreasing (Panels (b) and (d) of Figure 17, respectively). Though the decrease in the low-skilled labor share is larger than the increase in the high-skilled labor share, which leads the overall labor share to decline. The wage bills both become smoother largely due to the overall labor inputs becoming smoother which translates to somewhat smoother wages. This in turn leads to a smoother wage bill by type as the employment volatilities itself have not changed and implies the labor income risk of both types decline.

As $\bar{N}_H$ increases the relative price of high-skilled labor decreases relative to that of low-skilled labor. Since high-skilled labor and low-skilled labor are gross substitutes this leads to the low-skilled labor share to decline. Furthermore, more capital is used in production since there is more high-skilled labor due to capital-skill complementarities. In equilibrium this results in the relative price of capital to decline with respect to both types of labor which leads a further decrease in the low-skilled labor share and an increase in the high-skilled labor share. This is driven by low-skilled labor being a gross substitute and high-skilled labor being a gross complement to capital. This is similar to the mechanism of Karabarbounis and Neiman (2014), who attribute the declining relative price of capital and the gross substitutes relation between labor and capital to the declining labor share.

Finally, I examine how increasing $\bar{N}_H$ affects dividends and key financial moments. Figure 18 shows the comparative statics of $\bar{N}_H$ for dividends, investment, and various financial moments. Panel (a) of Figure 18 shows that dividends become more volatile with respect to $\bar{N}_H$. This is in part driven by an increase in profit volatility and in part driven by investment becoming smoother. Panels (b) and (c) of Figure 18 show that investment becomes smoother but the investment-to-output ratio increases, respectively. The effect of the former offsets the effect of the latter to make dividends more volatile. The investment-to-output ratio increases because of the increase in capital-skill complementarities as there is more high-skilled labor. However, this larger baseline results in investment growth becoming smoother. The smoother high-skilled wage bill offsets the more volatile dividend to make high-skilled consumption smoother resulting in a higher risk-free rate (Panel (d) of Figure 18). There is also the effect of greater risk-sharing as the wage bill takes more a greater proportion of high-skilled household's consumption, which increases the risk-free rate. This risk-sharing channel also feeds into supporting more volatile dividends. The combined effect of the greater cash-flow risks, greater risk sharing, and lower labor income risk leads to an ambiguous effect on the expected equity risk premium. While the expected equity return is increasing in $\bar{N}_H$ (Panel (e) of Figure 18), the effect of $\bar{N}_H$ on expected equity premium is non-monotonic (Panel (f) of Figure 18). For higher values of $\bar{N}_H$ the increased cash-flow risks dominate the increased risk-sharing; the reverse is true for lower value of $\bar{N}_H$.

Overall, the increase in high-skilled labor is an important driver of several key results. In addition to being able to help the model match key macroeconomic trends, increasing $\bar{N}_H$ also generates a higher expected equity risk premium given the calibration. The increase in risk-sharing also leads $\bar{N}_H$ to have an increasing effect on the risk-free rate. This is also a decreasing force on the risk premium, however, more high-skilled labor also results in more cash-flow risk that offsets the effects of increased risk-sharing and decreased labor income risk.

6 Conclusion

In this paper I examine how the increasing dependence on high-skilled labor in the macroeconomy is related to various recent trends such as a decreasing risk-free rate, increasing equity risk premium, and increasing stock market participation. I construct a dynamic general equilibrium model with capital-skill complementarities and calibrate it to match various macroeconomic moments across different time periods. This model is able to replicate several key trends in both macroeconomics and finance. The model generates these results by generating a smoother wage bill and lower labor share that leads to more volatile profits. Furthermore, as the investment-to-output ratio declines, investment is a less effective countercyclical weight to smooth dividends. Along with the previous forces, this leads to more volatile dividends and an increased equity premium. These forces also transmit through a general equilibrium response to the consumer's SDF. While the overall wage bill becomes smoother, the high-skilled wage bill becomes more volatile. Since only high-skilled households participate in financial markets, the higher labor income risk translates to a higher overall consumption risk for high-skilled households. This increases the volatility of their SDF, decreasing the risk-free rate and further increasing the equity risk premium.

High-skilled labor is complementary to capital which results in a smoother wage bill than that of low-skilled labor. A shift towards high-skilled labor thus leads to an overall smoother wage bill and lower labor share. However, the high-skilled wage bill itself becomes more volatile as the production technology depends more on high-skilled workers due to increased capital-skill complementarities. Since high-skilled households are the only participants in financial markets, these forces dominate the effects of increased risk sharing to generate the contradictory trends of higher stock market participation, lower risk-free rates, and higher equity risk premium that is observed in the data.

One limitation of my modeling strategy is that it relies on the production technology suddenly changing, but it does not explore the underlying cause of the technological change. This change itself may be a source of risk that drives many of these results. Furthermore, I model households as representative within each type and abstract away from between and within skill type trading for analytical and computational tractability. However, low-skilled households have some participation in the stock market and even within high-skilled households there is likely significant heterogeneity. Eisfeldt, Falato, and Xiaolan (2022) document that high-skilled workers are increasingly likely to be paid with stock or stock options by their employers, which are highly heterogeneous among these households and cannot be readily traded for significant periods of time. The interaction between the innovation driving the technological change as well as these household portfolio dynamics provide an interesting pathway for future research.

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Table 1: Baseline Model Full Time Series Calibration

Parameter	Description	Value
	Panel A. Households	
β	Subjective time discounting	0.994
γ	Relative risk aversion	7
ψ	Elasticity of intertemporal substitution	2
	Panel B. Firms	
α	Low-skilled intensity of production	0.473
λ	Capital intensity of inner nest	0.488
σ	Governs elasticity of substitution between N_L and (K, N_H)	0.401
ρ	Governs elasticity of substitution between N_H and K	−0.495
η	Returns to scale of labor	0.72
θ	Relative efficiency of high-skilled household	1
δ	Depreciation rate of capital	0.025
ν	Capital adjustment cost scale	6.7
ζ	Fixed operating cost scale	0.3462
$\bar{Z}$	Neutral productivity level factor	0.25
ρ_B	Persistence of financial leverage	0.9
κ	Target leverage ratio	0.37
	Panel C. Aggregate Productivity Growth and Labor Processes	
ρ_g	Persistence of aggregate productivity growth	0.8
$\bar{g}$	Unconditional mean aggregate productivity growth	0.005
σ_g	Conditional SD of aggregate productivity growth	0.01
$\bar{N}_H$	Unconditional mean proportion of high-skilled labor	0.2404
$\bar{N}_L$	Unconditional mean proportion of low-skilled labor	0.7596
σ_N	Common labor supply conditional SD scaling factor	0.0257
Σ_L	Relative low-skilled labor supply conditional SD	1.286

Note: This table contains the benchmark calibration for the baseline model (model with capital-skill complementarities). Panel A shows the parameters for household preferences. Panel B shows the parameters relevant to the firm such as the parameters of the production process and capital structure. Panel C shows the parameters of the processes for aggregate productivity growth and labor supply.

Table 2: Baseline Model Targeted Moments (All Calibrations)

	Benchmark		First Sub.		Second Sub.	
	Data (1)	Model (2)	Data (3)	Model (4)	Data (5)	Model (6)
High-Skilled Labor Share	0.2152	0.2196	0.1743	0.1766	0.2560	0.2592
Low-Skilled Labor Share	0.3675	0.3705	0.4155	0.4163	0.3195	0.3203
Overall Labor Share	0.5827	0.5901	0.5898	0.5929	0.5755	0.5795
Skilled Wage Premium	0.8834	0.8735	0.8119	0.8286	0.9548	0.9571
Investment-to-Output Ratio	0.1624	0.1635	0.1646	0.1656	0.1602	0.1609
Investment Growth Volatility	2.1072	2.0613	2.1072	2.0748	2.1072	2.0596
Market-to-Book Ratio	1.3300	1.3315	1.3300	1.3320	1.3300	1.3311
High-Skilled Employment Share	0.2404	0.2404	0.1883	0.1883	0.2926	0.2926
Low-Skilled Employment Share	0.7596	0.7596	0.8117	0.8117	0.7074	0.7074
High-Skilled Employment Growth Volatility	0.7724	0.7690	0.7724	0.7612	0.7724	0.7876
Low-Skilled Employment Growth Volatility	0.9956	0.9905	0.9956	0.9804	0.9956	1.0144

Note: This table shows the targeted moments for the baseline model's benchmark calibration. Column (1) contains the data moments and column (2) contains the model moments for the benchmark calibration. Columns (3) and (4) contains the data and model moments, respectively, for the first subperiod calibration. Columns (5) and (6) contains the data and model moments, respectively, for the second subperiod calibration. All columns at the annual level. All growth volatilities are relative to private GDP growth volatility. While the overall labor share is not a targeted moment since both in the model and in the data it is a perfect linear combination of the labor shares by skill, it is shown for convenience.

Table 3: Simple Model Benchmark Calibration

Parameter	Description	Value
α	Capital intensity of production	0.183
σ	Governs elasticity of substitution between N and K	-0.111
η	Returns to scale	0.71
ν	Capital adjustment cost scale	7.2
ζ	Fixed operating cost scale	0.03619
$\bar{N}$	Unconditional mean labor supply	1
σ_N	Conditional SD of labor supply	0.027

Note: This table contains the calibration for the simple model (model with homogeneous labor). This table only shows the parameters that are different from the baseline model's calibration. The only parameter that changes within the calibrations for the simple model is the participation parameter τ.

Table 4: Simple Model Calibration Targeted Moments (All Calibrations)

		Model		
	Data (1)	Benchmark (2)	First Sub. (3)	Second Sub. (4)
Overall Labor Share	0.5827	0.5849	0.5850	0.5850
Investment-to-Output Ratio	0.1624	0.1629	0.1632	0.1631
Investment Growth Volatility	2.1072	2.0858	2.0940	2.0912
Market-to-Book Ratio	1.3300	1.3336	1.3262	1.3281
Overall Employment Growth Volatility	0.8472	0.8467	0.8466	0.8467

Note: This table shows the targeted moments for the simple model's calibrations. Column (1) contains the data moments and columns (2) to (4) contains the model moments for. the benchmark, first subperiod, second subperiod calibrations, respectively. Both are at the annual level. All growth volatilities are relative to private GDP growth volatility.

Table 5: Baseline Model Benchmark Calibration Macroeconomic Moments

	Data			Model		
	$\sigma(g_x)/\sigma(g_y)$ (1)	$\rho(g_x, g_y)$ (2)	AC(g_x) (3)	$\sigma(g_x)/\sigma(g_y)$ (4)	$\rho(g_x, g_y)$ (5)	AC(g_x) (6)
GDP	1.000	1.000	0.230	1.000	1.000	0.367
Consumption	0.767	0.876	0.412	0.990	1.000	0.372
Investment	2.107	0.691	0.303	2.061	0.948	0.219
Employment	0.847	0.776	0.370	0.937	0.608	-0.137
Wages	0.636	0.178	0.147	0.853	0.445	0.560
Wage Bill	0.914	0.860	0.446	0.959	1.000	0.377
Labor Share	0.509	-0.428	0.117	0.046	-0.888	0.292
High-Skilled Employment	0.772	0.428	0.542	0.769	0.609	-0.137
High-Skilled Wages	0.727	0.286	0.134	0.786	0.347	0.549
High-Skilled Wage Bill	0.998	0.553	0.481	0.762	0.982	0.499
High-Skilled Labor Share	0.960	-0.505	0.160	0.286	-0.865	0.177
Low-Skilled Employment	0.996	0.768	0.312	0.990	0.608	-0.137
Low-Skilled Wages	0.648	0.217	0.152	0.883	0.534	0.591
Low-Skilled Wage Bill	1.037	0.885	0.391	1.085	0.999	0.330
Low-Skilled Labor Share	0.478	-0.146	0.116	0.098	0.834	0.092

Note: This table presents the macroeconomic moments for the baseline model's benchmark calibration and compares them to the data. All figures are at the annual level. The variable g_x denotes the growth rate of x. Columns (1) to (3) present the relative growth volatility to private GDP growth volatility, correlation with private GDP growth, and one year autocorrelation in the data, respectively. Columns (4) to (6) show the same but for the model. The data are from 1979 to 2020, unless otherwise specified. Private GDP growth volatility is 4.49% from 1945 to 2020. The output growth volatility in this calibration is $\sigma(g_y) = 4.61\%$.

Table 6: Profits and Dividends Moments

	Profits			Unlevered Dividends		
	$\sigma(g_\pi)$ (1)	$\rho(g_\pi, g_y)$ (2)	$\text{AC}(g_\pi)$ (3)	$\sigma(g_d)$ (4)	$\rho(g_d, g_y)$ (5)	$\text{AC}(g_d)$ (6)
Panel A. Full Time Series (1979 to 2020)						
Data	10.305	0.525	-0.045	34.425	0.153	-0.357
Model with Capital-Skill Complementarities	8.837	0.962	0.240	6.364	0.990	0.343
Model with Homogeneous Labor	8.252	0.960	0.268	4.754	0.996	0.406
Panel B. First Subperiod (1979 to 1999)						
Data	9.977	0.558	0.068	28.943	0.195	-0.066
Model with Capital-Skill Complementarities	8.695	0.964	0.236	5.369	0.997	0.368
Model with Homogeneous Labor	8.264	0.960	0.268	4.657	0.999	0.397
Panel C. Second Subperiod (2000 to 2020)						
Data	10.853	0.516	-0.178	39.676	0.136	-0.517
Model with Capital-Skill Complementarities	8.980	0.958	0.242	7.602	0.975	0.317
Model with Homogeneous Labor	8.261	0.960	0.268	4.692	0.998	0.398

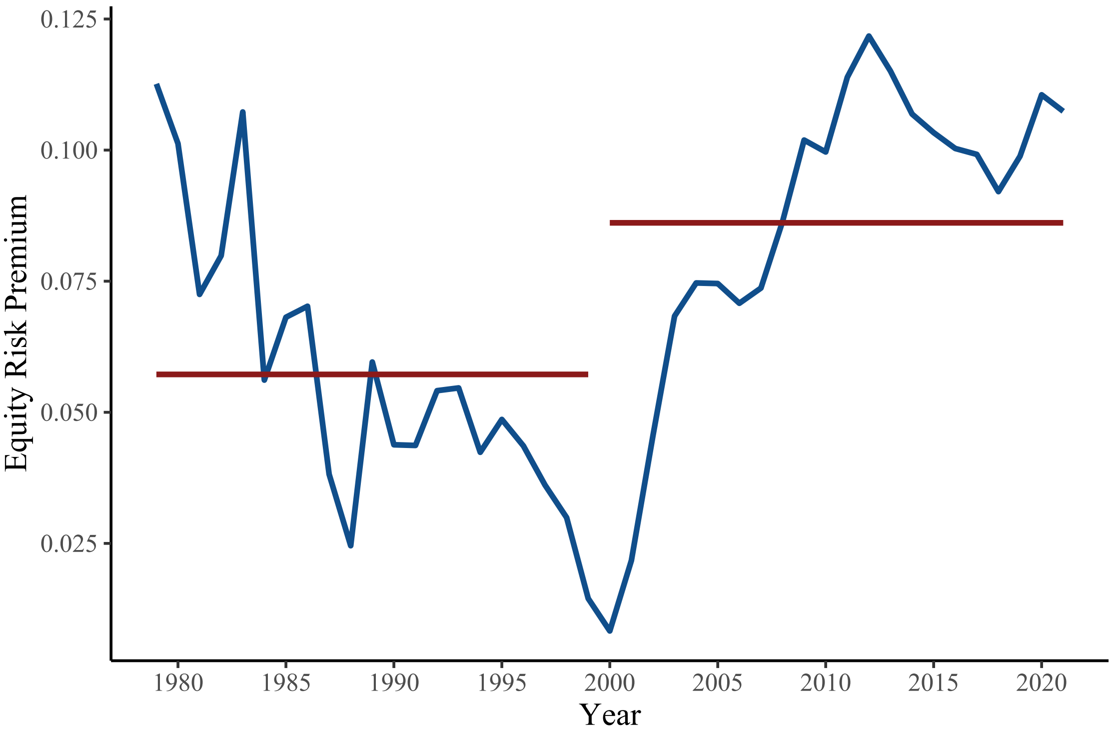
Note: This table shows the profits and dividends moments. All moments are at the annual level. Columns (1) to (3) show the growth volatility of profit in percent, the correlation between profit growth and private output growth, and the one year autocorrelation of profit growth, respectively. Columns (4) to (6) show the analagous statistics for unlevered dividends. The data on profits are from Compustat from 1979 to 2020 for all U.S. firms. Firms with SIC codes 6000 to 6700 and 9000+ are excluded. Profits are defined as sales less COGs and SGA. Dividends are computed have taking the definition of profits and substracting the first differene of gross PPE. This is to better match the definition of dividends in the model with the data. All nominal figures are deflated with CPI.

Table 7: Financial Moments

	$\mathbb{E}[r_f]$ (1)	$\sigma(r_f)$ (2)	$\mathbb{E}[r_E]$ (3)	$\sigma(r_E)$ (4)	SR(r_E) (5)	$\mathbb{E}[r_{EP}]$ (6)	$\sigma(r_{EP})$ (7)
Panel A. Full Time Series (1979 to 2020)							
Data	1.331	2.376	8.448	16.260	0.438	7.117	16.155
Model with Capital-Skill Complementarities	1.558	0.633	5.950	9.841	0.446	4.392	9.822
Model with Homogeneous Labor	1.431	0.580	5.734	9.614	0.448	4.303	9.601
Exchange Model	2.101	1.586	7.024	9.399	0.524	5.310	9.181
Panel B. First Subperiod (1979 to 1999)							
Data	2.976	2.017	8.698	13.098	0.437	5.723	13.159
Model with Capital-Skill Complementarities	1.639	0.635	5.733	9.264	0.442	4.094	9.239
Model with Homogeneous Labor	1.407	0.587	5.711	9.592	0.449	4.304	9.579
Exchange Model	1.925	1.391	7.014	9.699	0.525	5.489	9.536
Panel C. Second Subperiod (2000 to 2020)							
Data	-0.314	1.350	8.198	18.633	0.457	8.512	18.829
Model with Capital-Skill Complementarities	1.510	0.622	6.144	10.415	0.445	4.634	10.402
Model with Homogeneous Labor	1.414	0.586	5.717	9.597	0.448	4.303	9.584
Exchange Model	1.988	1.460	7.017	9.590	0.524	5.425	9.409

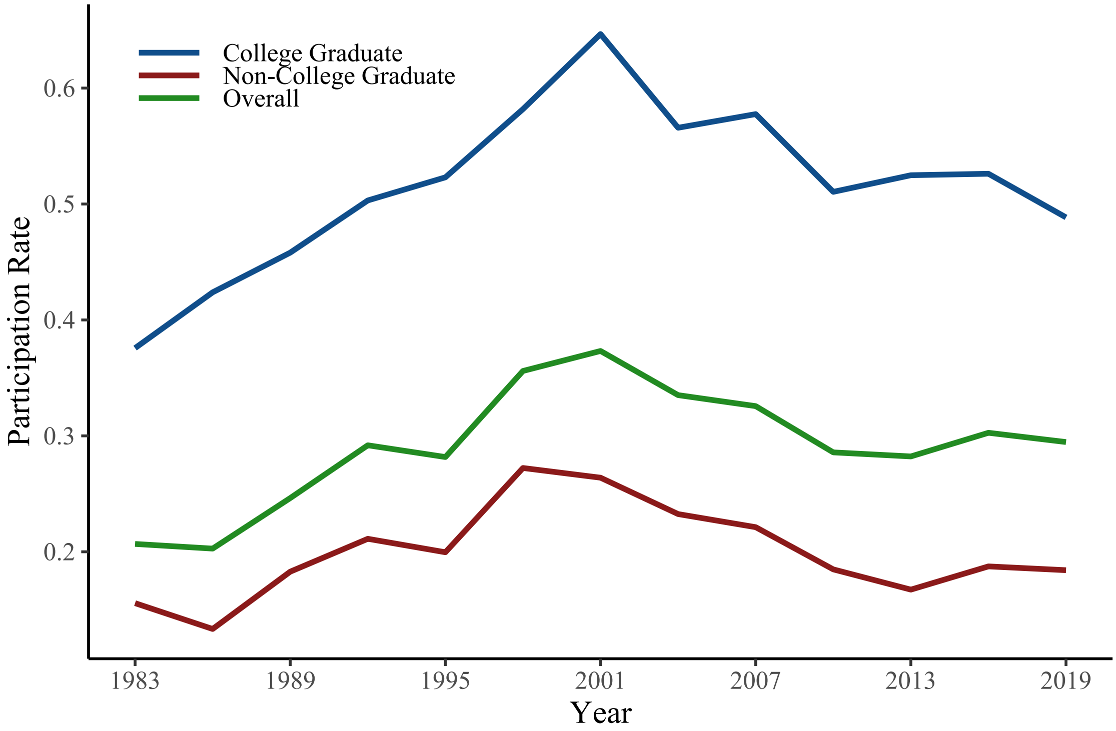
Note: This table shows the financial moments in the data and the model across various date ranges. All figures are at the annual level. Columns (1) and (2) show the mean and volatility of the risk-free rate. Columns (3) and (4) show the mean and volatility of equity returns. Column (5) shows the Sharpe ratio. Columns (6) and (7) show the mean and volatility of the equity premium. Panel A shows the moments for the full time series. Panel B shows the moments for the first subperiod (1979 to 1999) and the Panel C shows the moments for the second subperiod (2000 to 2020). For the data, I use the one year ahead expected equity premium estimates from Duarte and Rosa (2015) and the yield of the constant 1-year maturity U.S. Treasury Bill as the proxy for the risk-free rate. The equity return in the data is the implied one year ahead expected equity premium plus the proxy of the risk-free rate. I use the realized returns of the total market index from CRSP to estimate the volatility of equity returns and the equity premium. All returns are deflated with CPI.

Figure 1: Equity Risk Premium Estimate



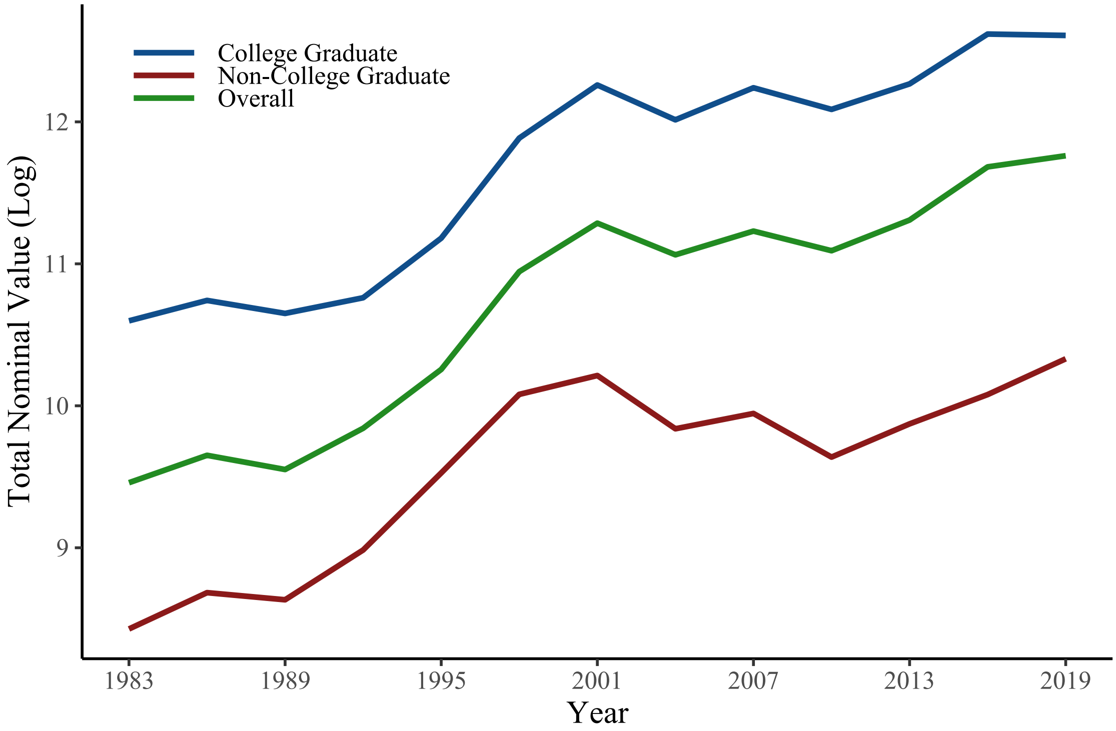
Note: This figure shows the one-year ahead equity risk premium estimate of Duarte and Rosa (2015) from 1979 to 2020. The time series average is 7.117%. The average for 1979 to 1999 is 5.723% and the average for 2000 to 2020 is 8.512%. The averages by subperiod are denoted in the figure by the two horizontal red lines.

Figure 2: Stock Market Participation



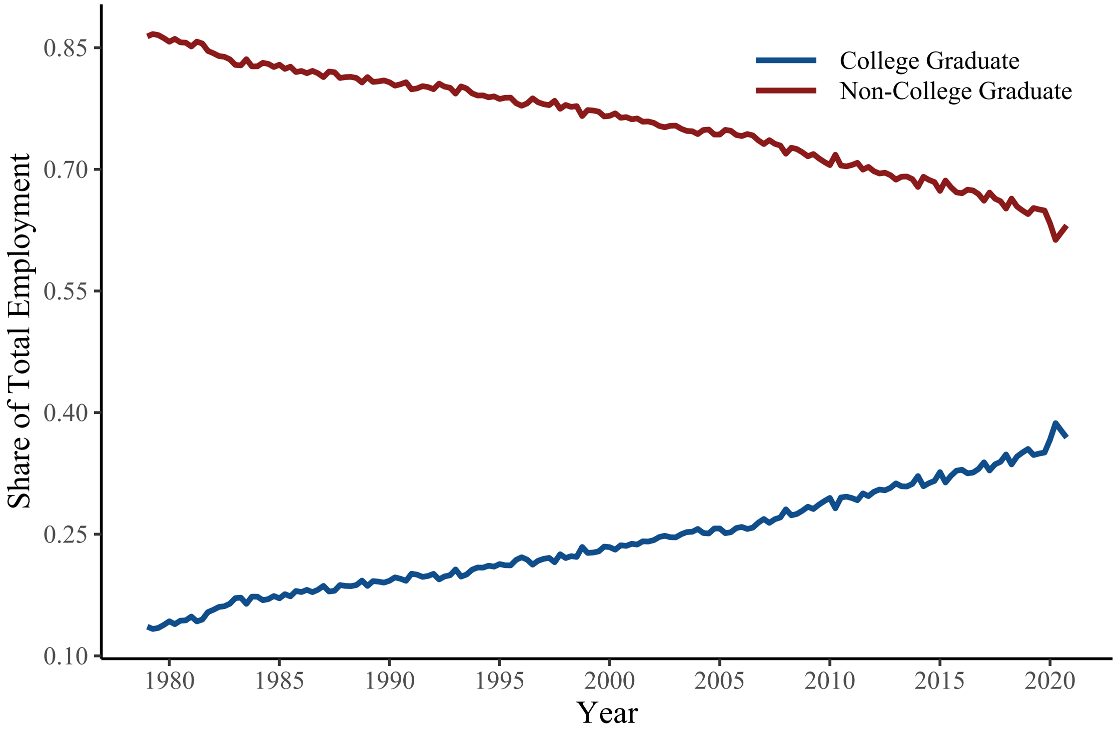
Note: This figure shows stock market participation in the United States by education and in the aggregate from 1983 to 2019 at the household level. The blue line shows the time series for households headed by a college graduate. The red line shows the time series for households headed by a non-college graduate. The green line shows the time series for all households. The data are from the Survey of Consumer Finances (SCF). A college graduate is defined an individual who has completed at least a four-year bachelor's degree. Stock market participation is defined as holding strictly positive wealth in the stock market, which includes direct holdings, mutual fund holdings, and holdings in tax-advantaged retirements.

Figure 3: Stock Market Wealth per Household



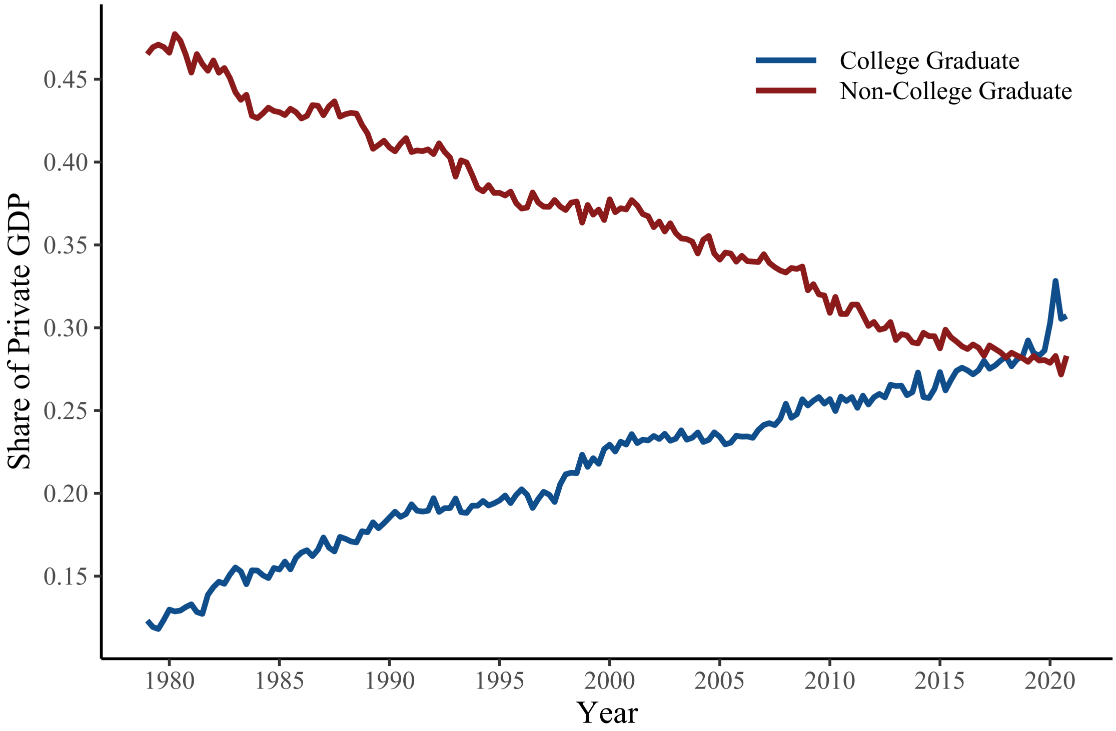
Note: This figure shows the average household wealth in the stock market by education and overall from 1983 to 2019. The blue line shows the time series for households headed by a college graduate. The red line shows the time series for households headed by a non-college graduate. The green line shows the time series for all households. The data are from the Survey of Consumer Finances (SCF). A college graduate is defined an individual who has completed at least a four-year Bachelor's degree. Stock market participation is defined as holding strictly positive wealth in the stock market.

Figure 4: Employment Share by Education



Note: This figure shows the employment share by education from 1979 to 2020 at a quarterly frequency. The blue line shows the share of college graduates and the red line shows the share of non-college graduates. The data are from the Bureau of Labor Statistics (BLS) Employment Situation reports and Consumer Population Survey (CPS). A college graduate is defined as an individual who has completed at least 16 years of education. See Section 2 and Appendix B for more information on how the CPS is used to generate this series.

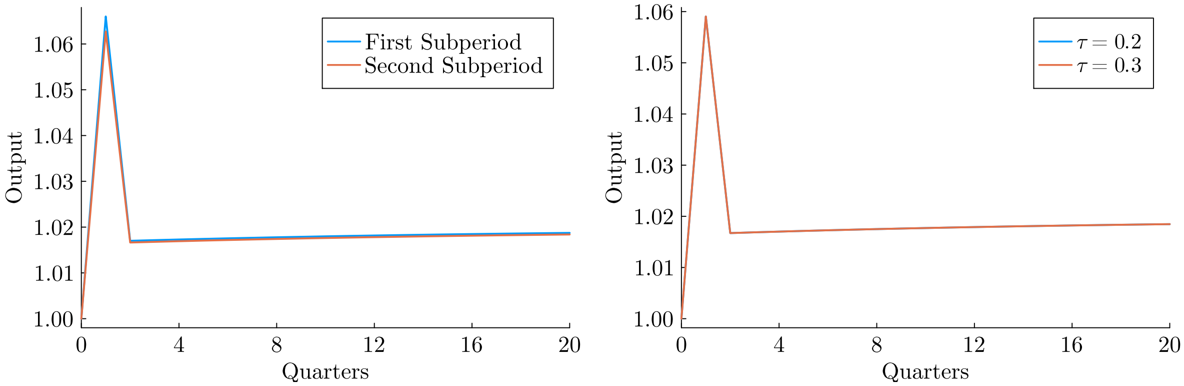
Figure 5: Labor Shares by Education



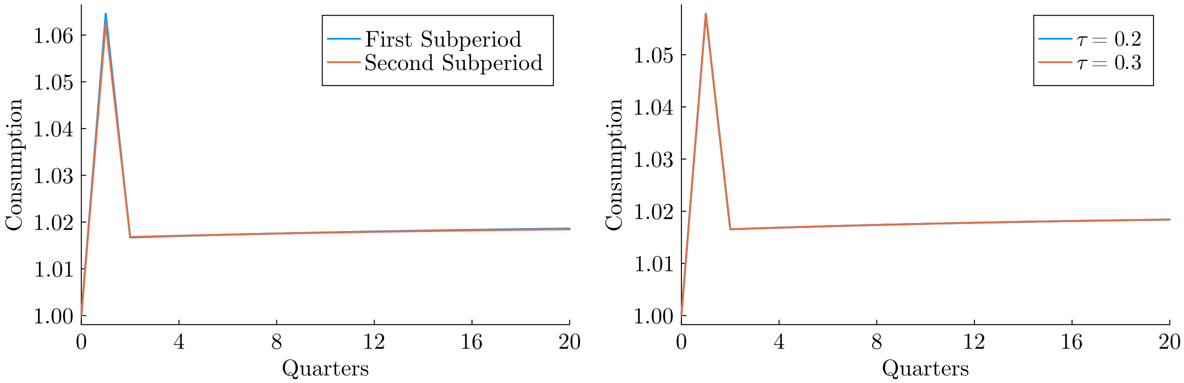
Note: This figure shows the labor shares by education from 1979 to 2020 at a quarterly frequency. Note that this definition of the labor share is defined as the wage bill to employees in the private sector (by education) over private GDP. The blue line is the high-skilled labor share and the red line is low-skilled labor share. The data are from the CPS and BEA. See Section 2 and Appendix B for more details on the construction of these variables.

Figure 6: Impulse Response Functions – Output and Consumption

(a) Output (Baseline Model) (b) Output (Simple Model)

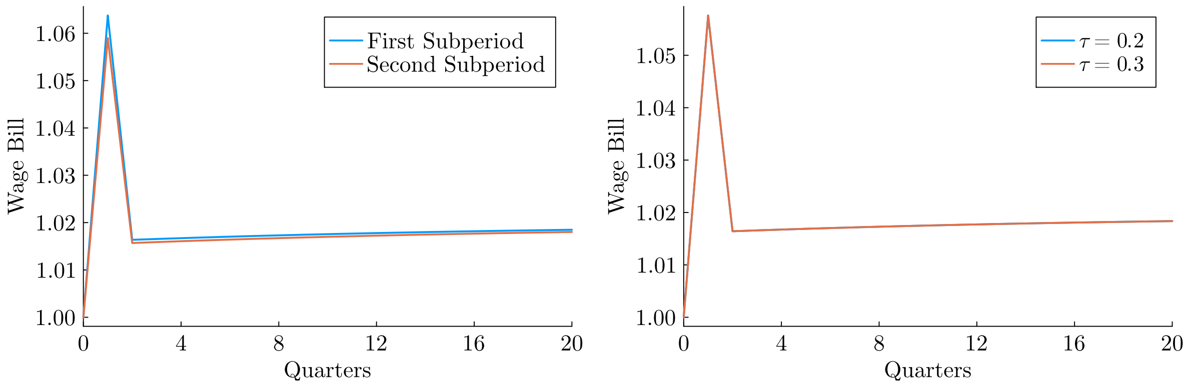


(c) Consumption (Baseline Model) (d) Consumption (Simple Model)



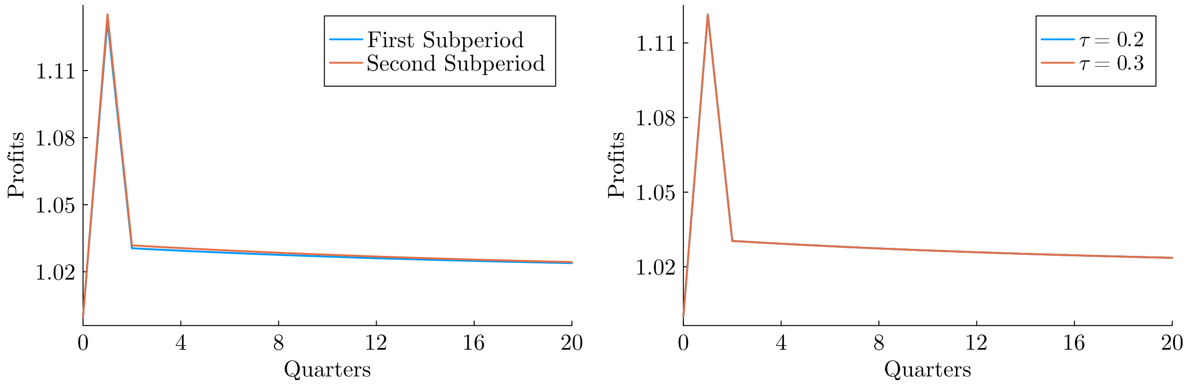
Note: This figure presents the impulse response functions for both the baseline model and simple model for output and consumption. Panels (a) and (c) show impulse response functions of output and consumption for the baseline model. Panels (b) and (d) show the same but for the simple model. The blue line shows represents the first subperiod calibration (labelled $\tau = 0.2$ for the simple model) and the red line represents the second subperiod calibration (labelled $\tau = 0.3$ for the simple model) across all panels. The shock is a positive aggregate productivity growth shock that lasts for one quarter and returns to the steady state value. The steady state is defined at the middle grid point of the discretized aggregate productivity growth process. The shock takes the aggregate productivity growth rate two points above the middle grid point. The x-axis is quarters after the shock and y-axis is the model quantity relative to the steady state value at the same quarter.

Figure 7: Impulse Response Functions – Wage Bill, Profits, and Dividends



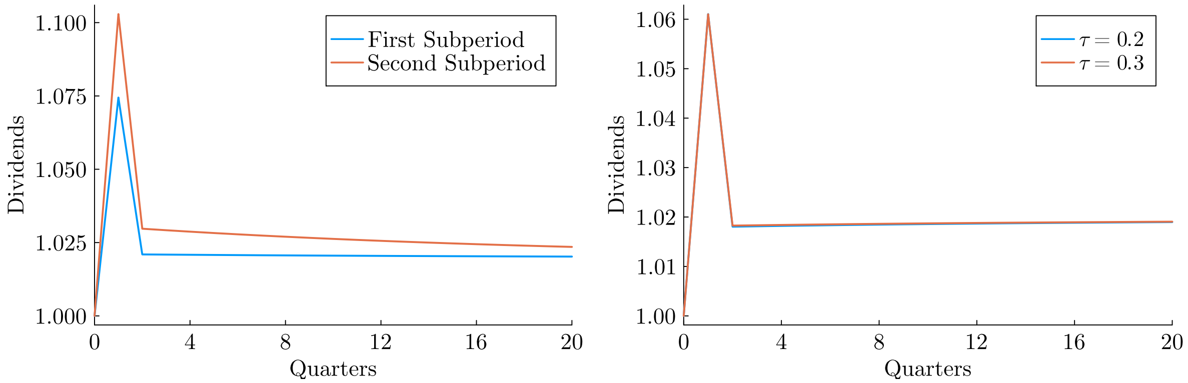
(a) Wage Bill (Baseline Model)

(b) Wage Bill (Simple Model)



(c) Profits (Baseline Model)

(d) Profits (Simple Model)

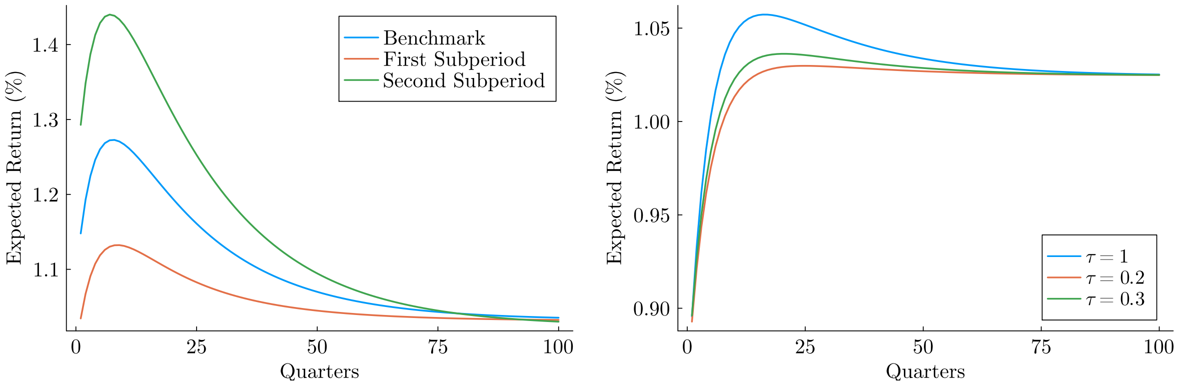


(e) Dividends (Baseline Model)

(f) Dividends (Simple Model)

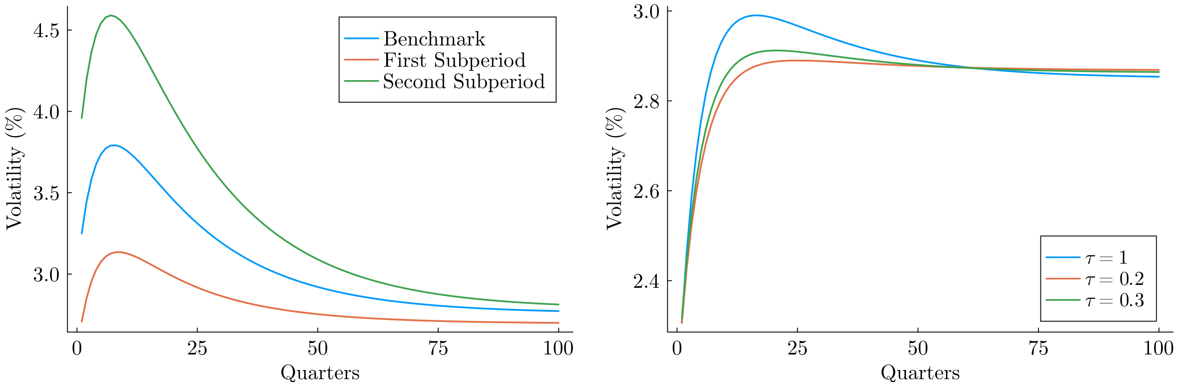
Note: This figure presents the impulse response functions for both the baseline model and simple model for the wage bill, profits, and dividends. Panels (a), (c), and (e) show impulse response functions of the wage bill, profits, and dividends for the baseline model. Panels (b), (d), and (f) show the same but for the simple model. The blue line shows represents the first subperiod calibration (labelled $\tau = 0.2$ for the simple model) and the red line represents the second subperiod calibration (labelled $\tau = 0.3$ for the simple model) across all panels. The shock is a positive aggregate productivity growth shock that lasts for four quarters and returns to the steady state value. The x-axis is quarters after the shock and y-axis is the model quantity relative to the steady state value at the same quarter. The steady state is defined at the middle grid point of the discretized aggregate productivity growth process.

Figure 8: Dividend Term Structure



(a) Expected Returns (Baseline Model)

(b) Expected Returns (Simple Model)

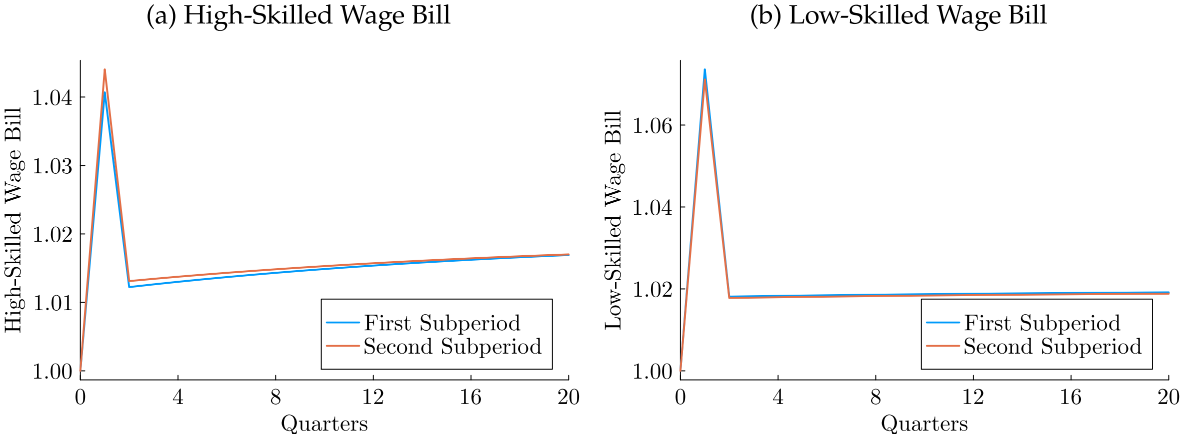


(c) Return Volatility (Baseline Model)

(d) Return Volatility (Simple Model)

Note: This figure shows the dividend term structure's expected return and return volatility for both the baseline and simple models. All figures are at the quarterly level and show the term structure up to 100 quarters (25 years). Panels (a) and (c) shows the expected return and return volatility of the dividend term structure of the baseline model, respectively. Panels (b) and (d) shows the expected return and return volatility of the dividend term structure of the simple model, respectively. Across all panels, the blue line corresponds to the benchmark calibration (labelled $\tau = 1$ for the simple model), the red line corresponds to the first subperiod calibration (labelled $\tau = 0.2$ for the simple model), and the green line corresponds to the second subperiod calibration (labelled $\tau = 0.3$ for the simple model).

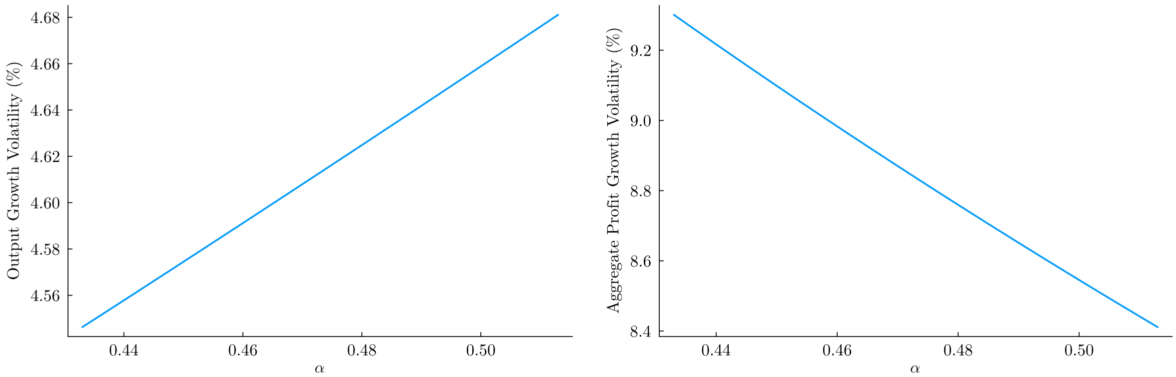
Figure 9: Impulse Response Functions – High-Skilled Wage Bill and Low-Skilled Wage Bill


(a) High-Skilled Wage Bill

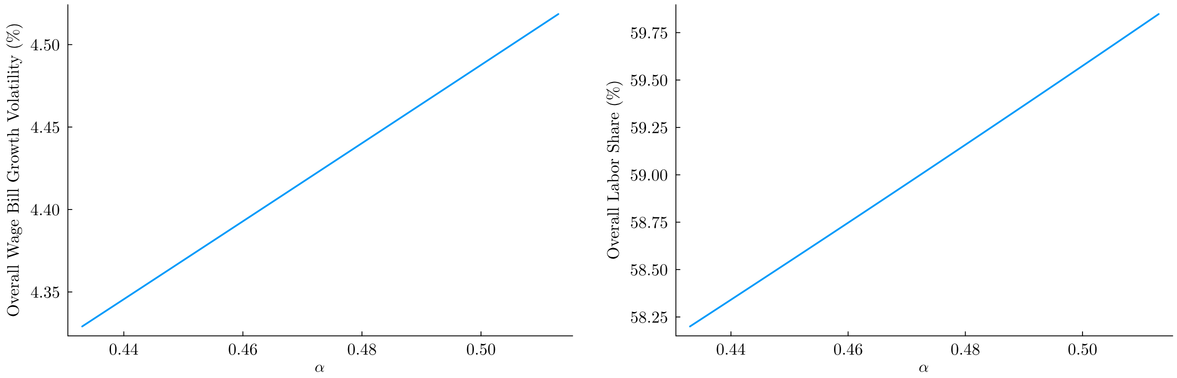
(b) Low-Skilled Wage Bill

Note: This figure presents the impulse response functions for for the high-skilled wage bill and low-skilled wage bill for the baseline model only. Panel (a) shows the impulse response function for the high-skilled wage bill. Panels (b) shows the same but for the low-skilled wage bill. The blue line shows represents the first subperiod calibration and the red line represents the second subperiod calibration across all panels. The shock is a positive aggregate productivity growth shock that lasts for four quarters and returns to the steady state value. The x-axis is quarters after the shock and y-axis is the model quantity relative to the steady state value at the same quarter. The steady state is defined at the middle grid point of the discretized aggregate productivity growth process.

Figure 10: Comparative Statics of α – Profit Volatility and Determinants


(a) Output Growth Volatility

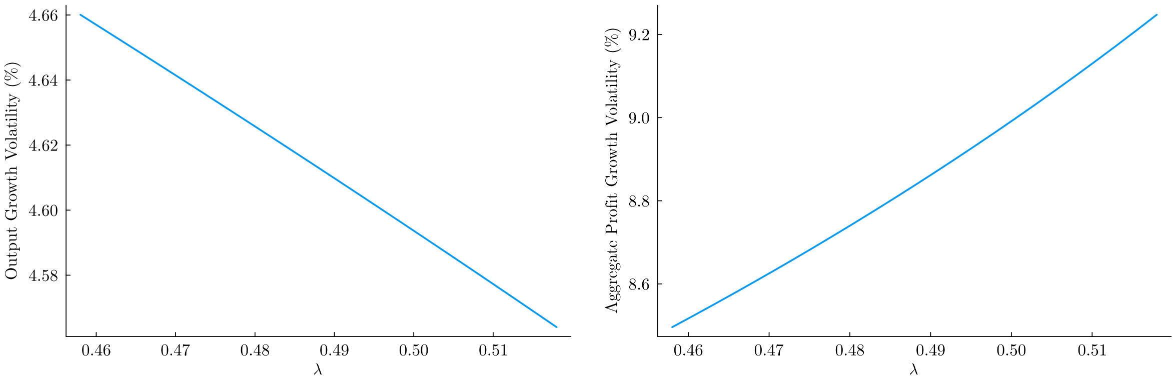
(b) Profit Growth Volatility


(c) Overall Wage Bill Growth Volatility

(d) Overall Labor Share

Note: This figure shows the comparative statics of α for profit volatility and its key immediate determinants. Panel (a) shows the volatility of output growth. Panel (b) shows the volatility of profit growth. Panel (c) shows the volatility of the overall wage bill. Panel (d) shows the overall labor share.

Figure 11: Comparative Statics of λ – Profit Volatility and Determinants

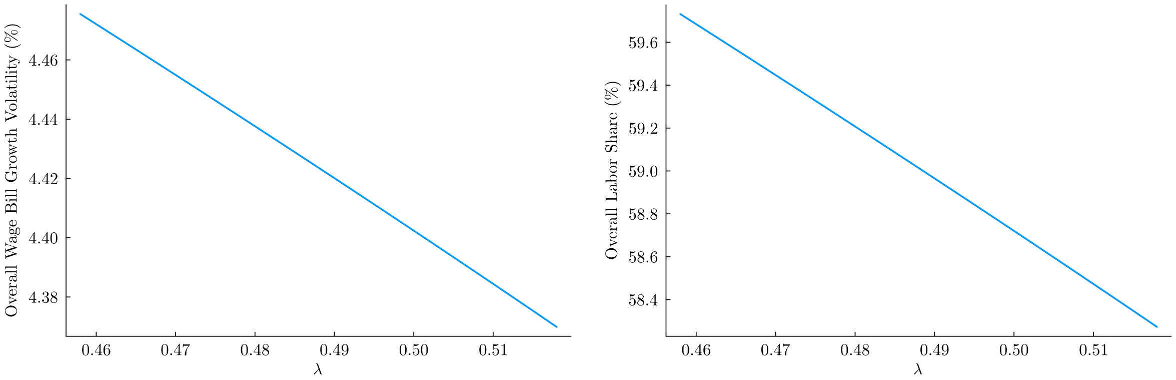


(a) Output Growth Volatility

(b) Profit Growth Volatility

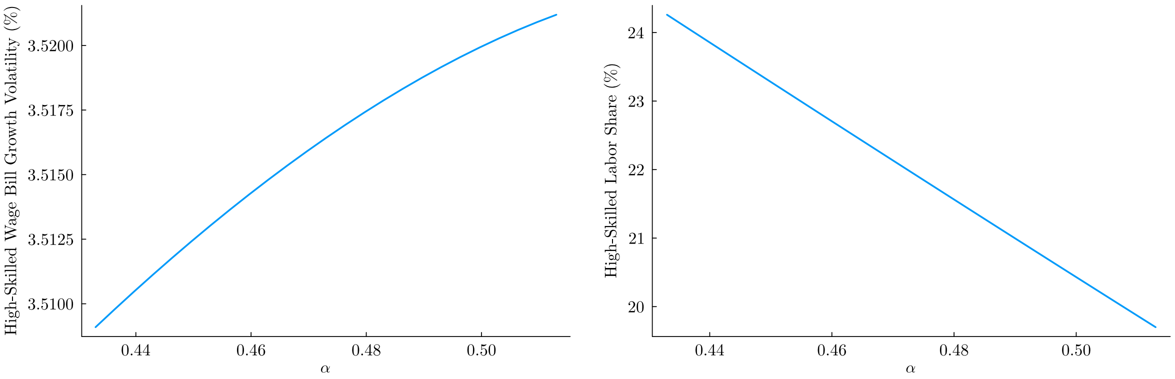
(c) Overall Wage Bill Growth Volatility

(d) Overall Labor Share



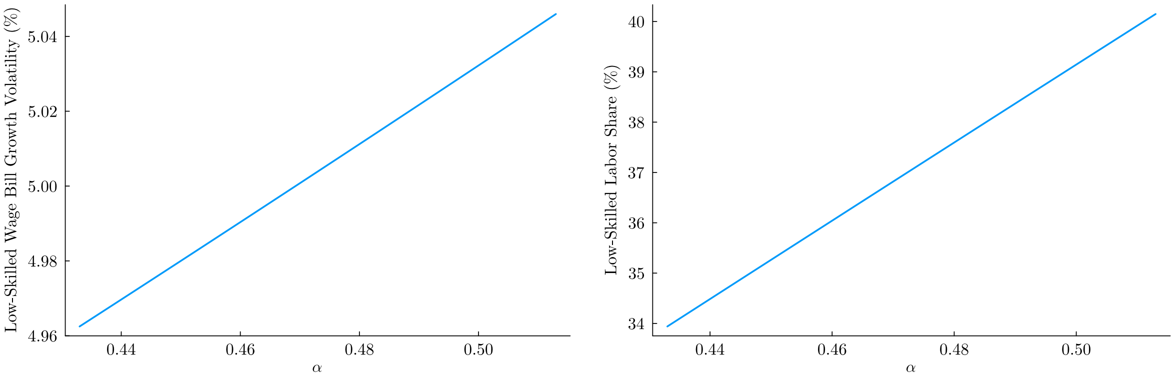
Note: This figure shows the comparative statics of λ for profit volatility and its key immediate determinants. Panel (a) shows the volatility of output growth. Panel (b) shows the volatility of profit growth. Panel (c) shows the volatility of the overall wage bill. Panel (d) shows the overall labor share.

Figure 12: Comparative Statics of α – Wage Bill and Labor Share by Skill Type



(a) High-Skilled Wage Bill Growth Volatility

(b) High-Skilled Labor Share

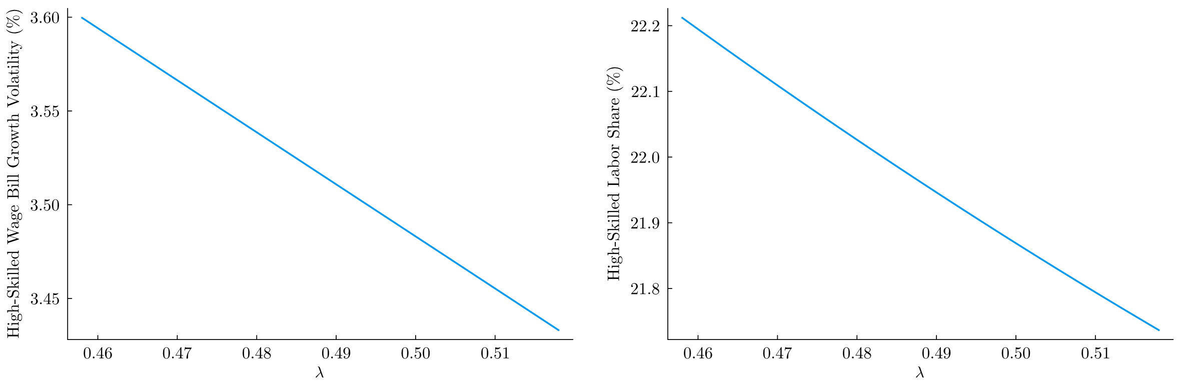


(c) Low-Skilled Wage Bill Growth Volatility

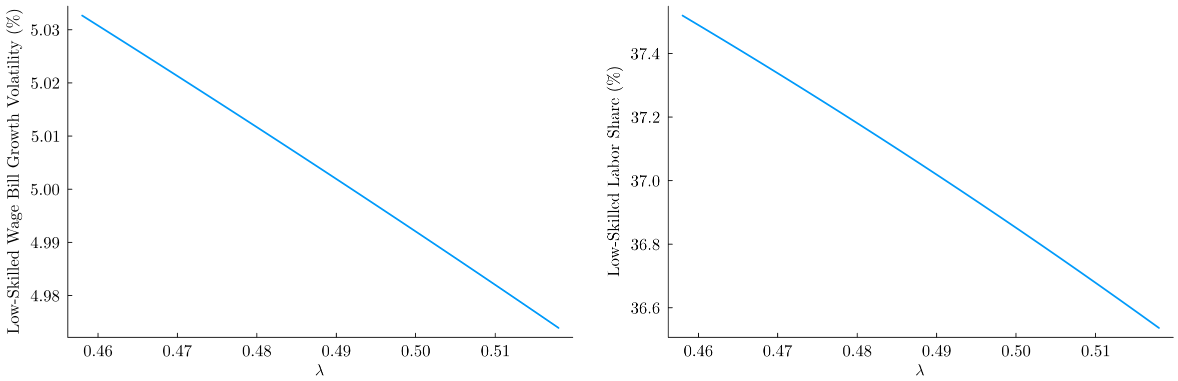
(d) Low-Skilled Labor Share

Note: This figure shows the comparative statics of α for the wage bill and labor share by skill type. Panels (a) and (b) show the volatility of high-skilled wage bill growth and the high-skilled labor share, respectively. Panels (c) and (d) show the volatility of low-skilled wage bill growth and the low-skilled labor share, respectively.

Figure 13: Comparative Statics of λ – Wage Bill and Labor Share by Skill Type


(a) High-Skilled Wage Bill Growth Volatility

(b) High-Skilled Labor Share


(c) Low-Skilled Wage Bill Growth Volatility

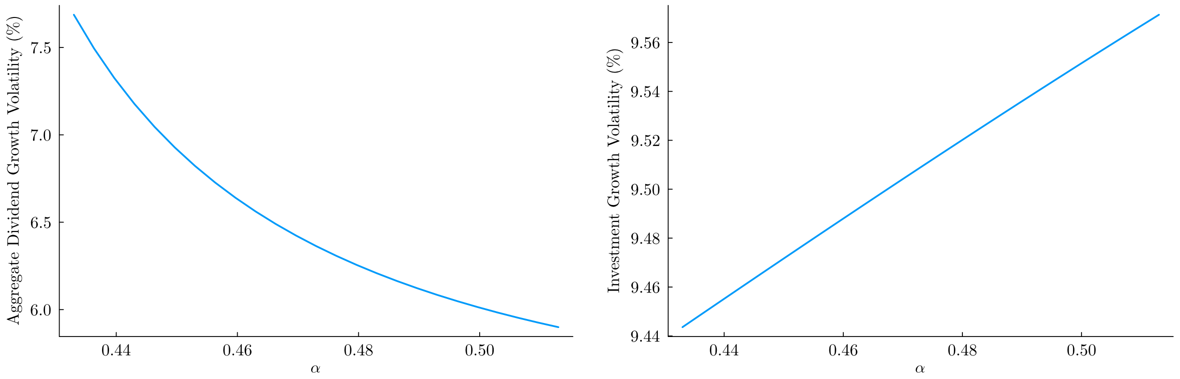
(d) Low-Skilled Labor Share

Note: This figure shows the comparative statics of λ for the wage bill and labor share by skill type. Panels (a) and (b) show the volatility of high-skilled wage bill growth and the high-skilled labor share, respectively. Panels (c) and (d) show the volatility of low-skilled wage bill growth and the low-skilled labor share, respectively.

Figure 14: Comparative Statics of α – Dividends, Investment, and Key Financial Moments

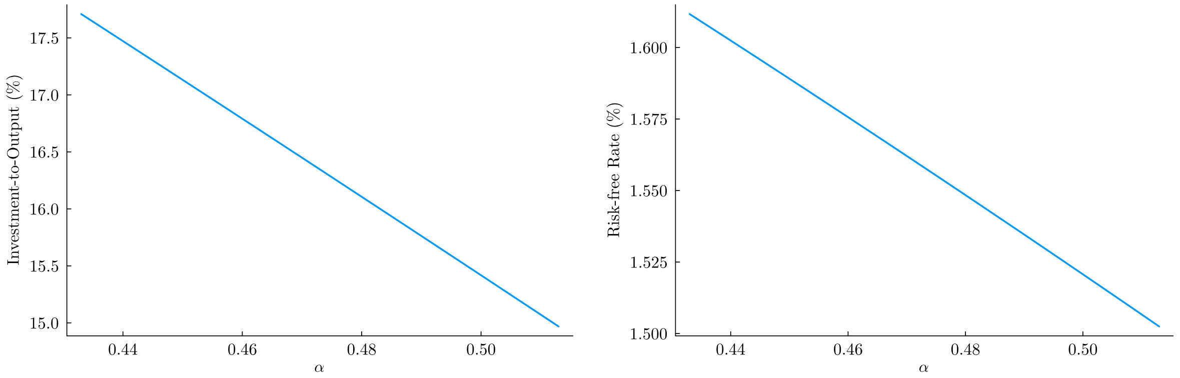
(a) Dividend Growth Volatility

(b) Investment



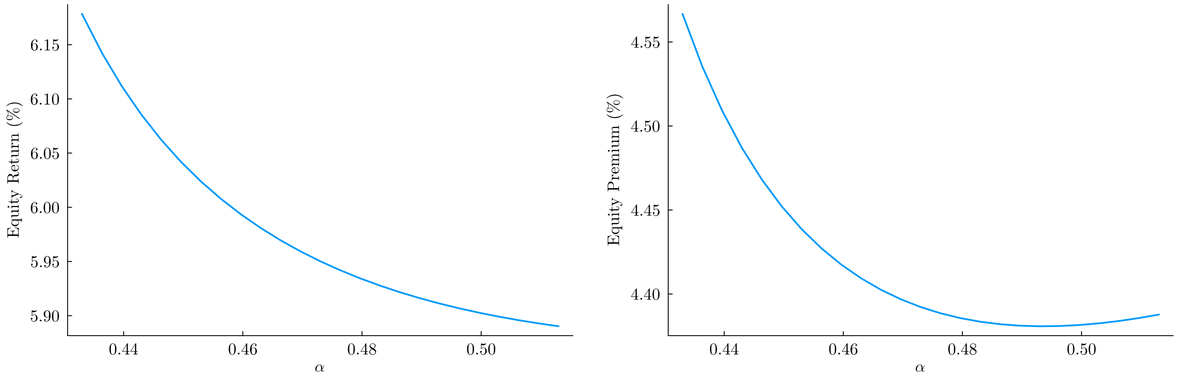
(c) Investment-to-Output Ratio

(d) Risk-free Rate



(e) Expected Equity Return

(f) Expected Equity Risk Premium

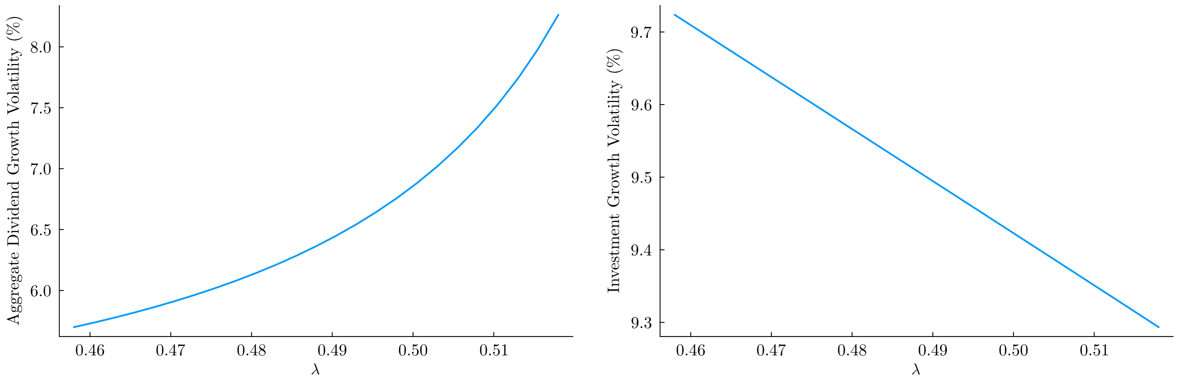


Note: This figure shows the comparative statics of α for dividends, investment, and various financial moments. Panel (a) shows the volatility of dividend growth. Panels (b) and (c) show the volatility of investment growth and investment-to-output ratio, respectively. Panel (d) shows the risk-free rate. Panels (e) and (f) show the expected equity return and expected equity premium, respectively.

Figure 15: Comparative Statics of λ – Dividends, Investment, and Key Financial Moments

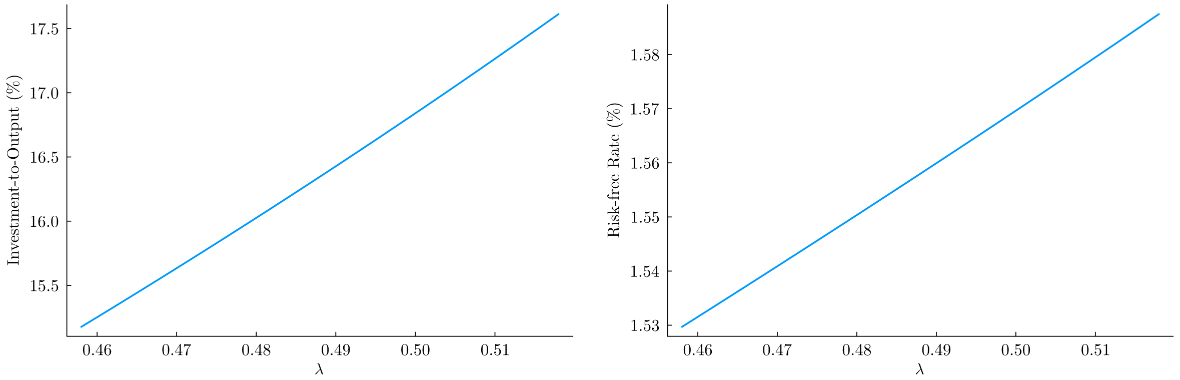
(a) Dividend Growth Volatility

(b) Investment



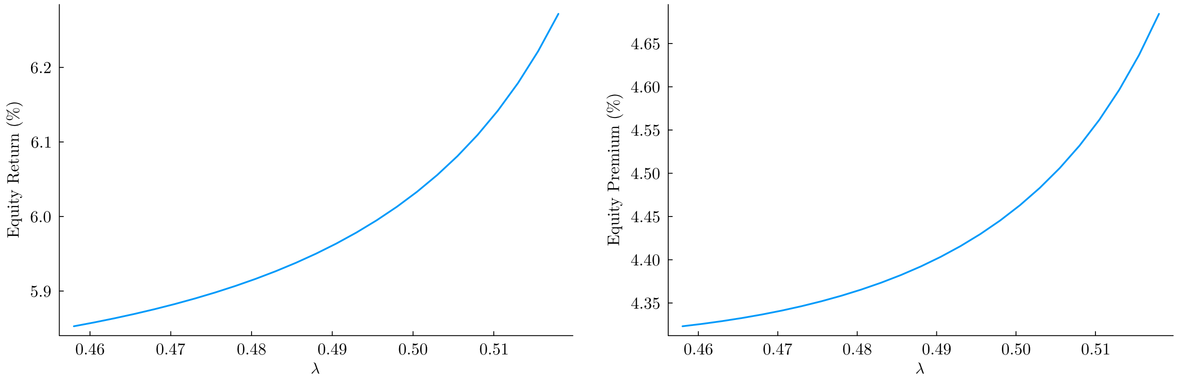
(c) Investment-to-Output Ratio

(d) Risk-free Rate



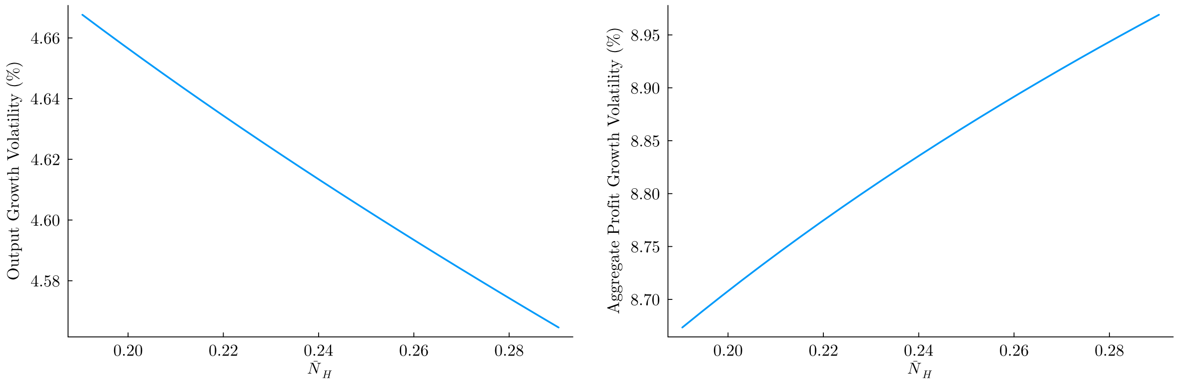
(e) Expected Equity Return

(f) Expected Equity Risk Premium



Note: This figure shows the comparative statics of λ for dividends, investment, and various financial moments. Panel (a) shows the volatility of dividend growth. Panels (b) and (c) show the volatility of investment growth and investment-to-output ratio, respectively. Panel (d) shows the risk-free rate. Panels (e) and (f) show the expected equity return and expected equity premium, respectively.

Figure 16: Comparative Statics of $\bar{N}_H$ – Profit Volatility and Determinants

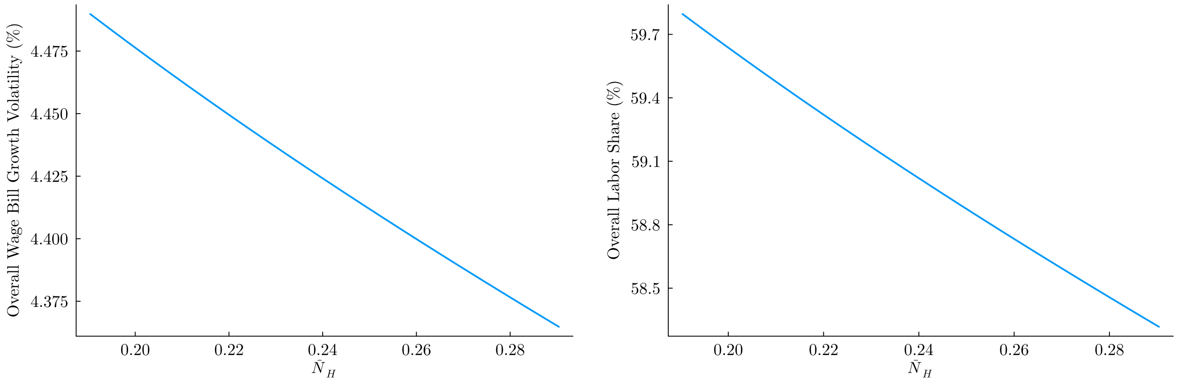


(a) Output Growth Volatility

(b) Profit Growth Volatility

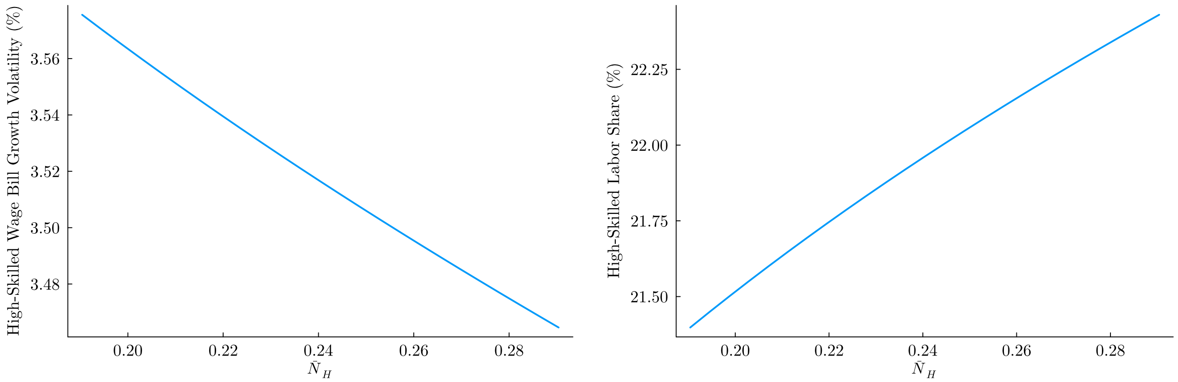
(c) Overall Wage Bill Growth Volatility

(d) Overall Labor Share



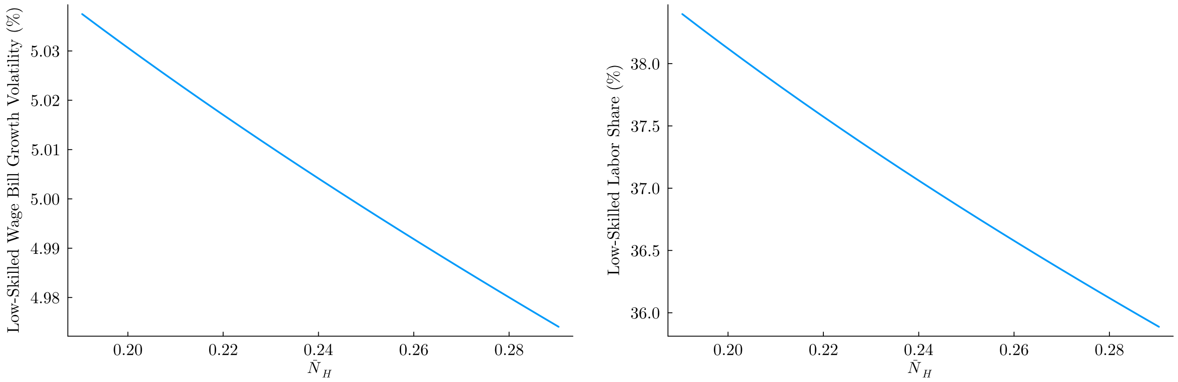
Note: This figure shows the comparative statics of $\bar{N}_H$ for profit volatility and its key immediate determinants. Panel (a) shows the volatility of output growth. Panel (b) shows the volatility of profit growth. Panel (c) shows the volatility of the overall wage bill. Panel (d) shows the overall labor share.

Figure 17: Comparative Statics of $\bar{N}_H$ – Wage Bill and Labor Share by Skill Type



(a) High-Skilled Wage Bill Growth Volatility

(b) High-Skilled Labor Share

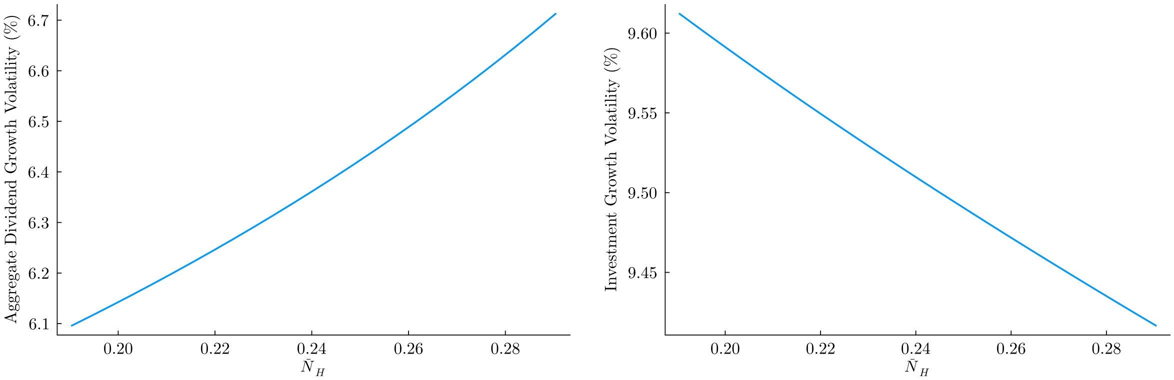


(c) Low-Skilled Wage Bill Growth Volatility

(d) Low-Skilled Labor Share

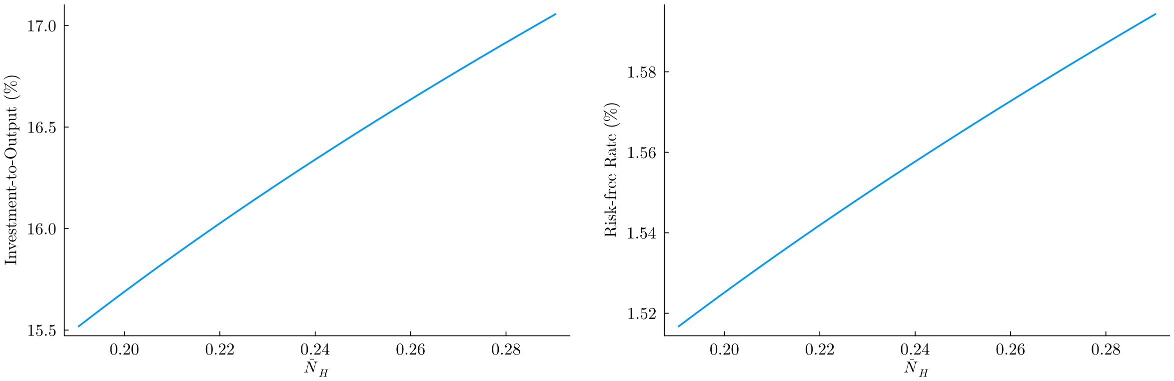
Note: This figure shows the comparative statics of $\bar{N}_H$ for the wage bill and labor share by skill type. Panels (a) and (b) show the volatility of high-skilled wage bill growth and the high-skilled labor share, respectively. Panels (c) and (d) show the volatility of low-skilled wage bill growth and the low-skilled labor share, respectively.

Figure 18: Comparative Statics of $\bar{N}_H$ – Dividends, Investment, and Key Financial Moments



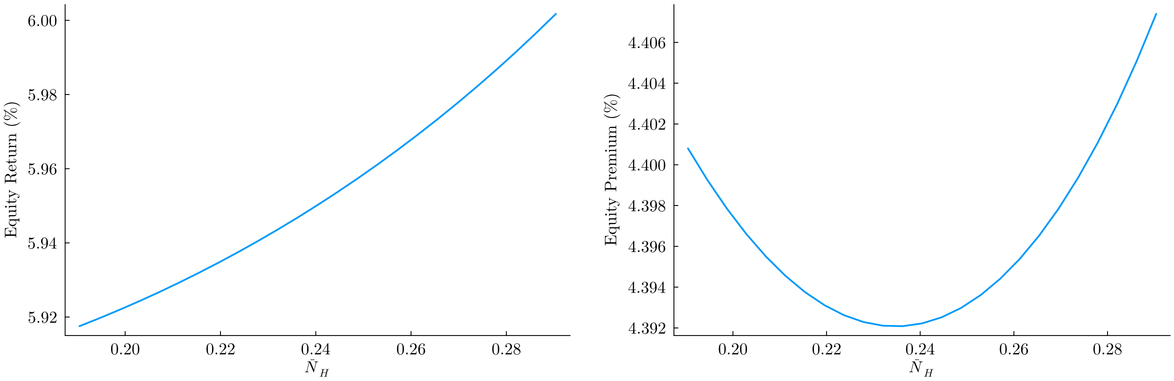
(a) Dividend Growth Volatility

(b) Investment



(c) Investment-to-Output Ratio

(d) Risk-free Rate



(e) Expected Equity Return

(f) Expected Equity Risk Premium

Note: This figure shows the comparative statics of $\bar{N}_H$ for dividends, investment, and various financial moments. Panel (a) shows the volatility of dividend growth. Panels (b) and (c) show the volatility of investment growth and investment-to-output ratio, respectively. Panel (d) shows the risk-free rate. Panels (e) and (f) show the expected equity return and expected equity premium, respectively.

A Additional Tables and Figures

Table A.1: Baseline Model First Subperiod Calibration

Parameter	Description	Value
	Panel A. Households	
β	Subjective time discounting	0.994
γ	Relative risk aversion	7
ψ	Elasticity of intertemporal substitution	2
	Panel B. Firms	
α	Low-skilled intensity of production	0.533
λ	Capital intensity of inner nest	0.567
σ	Governs elasticity of substitution between N_L and (K, N_H)	0.401
ρ	Governs elasticity of substitution between N_H and K	−0.495
η	Returns to scale of labor	0.72
θ	Relative efficiency of high-skilled household	1
δ	Depreciation rate of capital	0.025
ν	Capital adjustment cost scale	5.0
ζ	Fixed operating cost scale	0.03354
$\bar{Z}$	Neutral productivity level factor	0.25
ρ_B	Persistence of financial leverage	0.9
κ	Target leverage ratio	0.37
	Panel C. Aggregate Productivity Growth and Labor Processes	
ρ_g	Persistence of aggregate productivity growth	0.8
$\bar{g}$	Unconditional mean aggregate productivity growth	0.005
σ_g	Conditional SD of aggregate productivity growth	0.01
$\bar{N}_H$	Unconditional mean proportion of high-skilled labor	0.1883
$\bar{N}_L$	Unconditional mean proportion of low-skilled labor	0.8117
σ_N	Common labor supply conditional SD scaling factor	0.0257
Σ_L	Relative low-skilled labor supply conditional SD	1.286

Note: This table contains the first subperiod calibration for the baseline model (model with capital-skill complementarities). Panel A shows the parameters for household preferences. Panel B shows the parameters relevant to the firm such as the parameters of the production process and capital structure. Panel C shows the parameters of the processes for aggregate productivity growth and labor supply.

Table A.2: Baseline Model Second Subperiod Calibration

Parameter	Description	Value
	Panel A. Households	
β	Subjective time discounting	0.994
γ	Relative risk aversion	7
ψ	Elasticity of intertemporal substitution	2
	Panel B. Firms	
α	Low-skilled intensity of production	0.418
λ	Capital intensity of inner nest	0.424
σ	Governs elasticity of substitution between N_L and (K, N_H)	0.401
ρ	Governs elasticity of substitution between N_H and K	−0.495
η	Returns to scale of labor	0.71
θ	Relative efficiency of high-skilled household	1
δ	Depreciation rate of capital	0.025
ν	Capital adjustment cost scale	8.2
ζ	Fixed operating cost scale	0.03719
$\bar{Z}$	Neutral productivity level factor	0.25
ρ_B	Persistence of financial leverage	0.9
κ	Target leverage ratio	0.37
	Panel C. Aggregate Productivity Growth and Labor Processes	
ρ_g	Persistence of aggregate productivity growth	0.8
$\bar{g}$	Unconditional mean aggregate productivity growth	0.005
σ_g	Conditional SD of aggregate productivity growth	0.01
$\bar{N}_H$	Unconditional mean proportion of high-skilled labor	0.2926
$\bar{N}_L$	Unconditional mean proportion of low-skilled labor	0.7074
σ_N	Common labor supply conditional SD scaling factor	0.0257
Σ_L	Relative low-skilled labor supply conditional SD	1.286

Note: This table contains the second subperiod calibration for the baseline model (model with capital-skill complementarities). Panel A shows the parameters for household preferences. Panel B shows the parameters relevant to the firm such as the parameters of the production process and capital structure. Panel C shows the parameters of the processes for aggregate productivity growth and labor supply.

Table A.3: Baseline Model First Subperiod Calibration Macroeconomic Moments

	Data			Model		
	$\sigma(g_x)/\sigma(g_y)$ (1)	$\rho(g_x, g_y)$ (2)	AC(g_x) (3)	$\sigma(g_x)/\sigma(g_y)$ (4)	$\rho(g_x, g_y)$ (5)	AC(g_x) (6)
GDP	1.000	1.000	0.230	1.000	1.000	0.360
Consumption	0.767	0.876	0.412	0.982	1.000	0.366
Investment	2.107	0.691	0.303	2.075	0.948	0.215
Employment	0.847	0.776	0.370	0.939	0.615	-0.137
Wages	0.636	0.178	0.147	0.847	0.451	0.566
Wage Bill	0.914	0.860	0.446	0.969	1.000	0.366
Labor Share	0.509	-0.428	0.117	0.034	-0.890	0.318
High-Skilled Employment	0.772	0.428	0.542	0.761	0.615	-0.137
High-Skilled Wages	0.727	0.286	0.134	0.780	0.299	0.532
High-Skilled Wage Bill	0.998	0.553	0.481	0.729	0.974	0.519
High-Skilled Labor Share	0.960	-0.505	0.160	0.329	-0.868	0.177
Low-Skilled Employment	0.996	0.768	0.312	0.980	0.615	-0.137
Low-Skilled Wages	0.648	0.217	0.152	0.873	0.534	0.595
Low-Skilled Wage Bill	1.037	0.885	0.391	1.080	0.999	0.327
Low-Skilled Labor Share	0.478	-0.146	0.116	0.092	0.841	0.101

Note: This table presents the macroeconomic moments for the baseline model's first subperiod calibration and compares them to the data. All figures are at the annual level. The variable g_x denotes the growth rate of x. Columns (1) to (3) present the relative growth volatility to private GDP growth volatility, correlation with private GDP growth, and one year autocorrelation in the data, respectively. Columns (4) to (6) show the same but for the model. The data are from 1979 to 2020. Private GDP growth volatility is 4.49% from 1945 to 2020. The output growth volatility in this calibration is $\sigma(g_y) = 4.66\%$.

Table A.4: Baseline Model Second Subperiod Calibration Macroeconomic Moments

	Data			Model		
	$\sigma(g_x)/\sigma(g_y)$ (1)	$\rho(g_x, g_y)$ (2)	AC(g_x) (3)	$\sigma(g_x)/\sigma(g_y)$ (4)	$\rho(g_x, g_y)$ (5)	AC(g_x) (6)
GDP	1.000	1.000	0.230	1.000	1.000	0.374
Consumption	0.767	0.876	0.412	1.005	1.000	0.376
Investment	2.107	0.691	0.303	2.060	0.947	0.222
Employment	0.847	0.776	0.370	0.948	0.601	-0.137
Wages	0.636	0.178	0.147	0.865	0.428	0.546
Wage Bill	0.914	0.860	0.446	0.950	1.000	0.388
Labor Share	0.509	-0.428	0.117	0.056	-0.884	0.277
High-Skilled Employment	0.772	0.428	0.542	0.788	0.602	-0.137
High-Skilled Wages	0.727	0.286	0.134	0.800	0.372	0.552
High-Skilled Wage Bill	0.998	0.553	0.481	0.790	0.987	0.487
High-Skilled Labor Share	0.960	-0.505	0.160	0.251	-0.862	0.178
Low-Skilled Employment	0.996	0.768	0.312	1.014	0.601	-0.137
Low-Skilled Wages	0.648	0.217	0.152	0.898	0.521	0.579
Low-Skilled Wage Bill	1.037	0.885	0.391	1.089	0.998	0.334
Low-Skilled Labor Share	0.478	-0.146	0.116	0.104	0.825	0.083

Note: This table presents the macroeconomic moments for the baseline model's second subperiod calibration and compares them to the data. All figures are at the annual level. The variable g_x denotes the growth rate of x. Columns (1) to (3) present the relative growth volatility to private GDP growth volatility, correlation with private GDP growth, and one year autocorrelation in the data, respectively. Columns (4) to (6) show the same but for the model. The data are from 1979 to 2020. Private GDP growth volatility is 4.49% from 1945 to 2020. The output growth volatility in this calibration is $\sigma(g_y) = 4.50\%$.

Table A.5: Simple Model Benchmark Calibration Macroeconomic Moments

	Data			Model		
	$\sigma(g_x)/\sigma(g_y)$ (1)	$\rho(g_x, g_y)$ (2)	AC(g_x) (3)	$\sigma(g_x)/\sigma(g_y)$ (4)	$\rho(g_x, g_y)$ (5)	AC(g_x) (6)
GDP	1.000	1.000	0.230	1.000	1.000	0.405
Consumption	0.767	0.876	0.412	0.989	1.000	0.411
Investment	2.107	0.691	0.303	2.086	0.944	0.251
Employment	0.847	0.776	0.370	0.847	0.570	-0.137
Wages	0.636	0.178	0.147	0.865	0.568	0.607
Wage Bill	0.914	0.860	0.446	0.982	1.000	0.412
Labor Share	0.509	-0.428	0.117	0.021	-0.857	0.218

Note: This table presents the macroeconomic moments for the simple model's benchmark calibration and compares them to the data. All figures are at the annual level. The variable g_x denotes the growth rate of x. Columns (1) to (3) present the relative growth volatility to private GDP growth volatility, correlation with private GDP growth, and one year autocorrelation in the data, respectively. Columns (4) to (6) show the same but for the model. The data are from 1979 to 2020, unless otherwise specified. Private GDP growth volatility is 4.49% from 1945 to 2020. The output growth volatility in this calibration is $\sigma(g_y) = 4.40\%$.

Table A.6: Simple Model First Subperiod Calibration Macroeconomic Moments

	Data			Model		
	$\sigma(g_x)/\sigma(g_y)$ (1)	$\rho(g_x, g_y)$ (2)	$\text{AC}(g_x)$ (3)	$\sigma(g_x)/\sigma(g_y)$ (4)	$\rho(g_x, g_y)$ (5)	$\text{AC}(g_x)$ (6)
GDP	1.000	1.000	0.230	1.000	1.000	0.405
Consumption	0.767	0.876	0.412	0.987	1.000	0.411
Investment	2.107	0.691	0.303	2.094	0.943	0.252
Employment	0.847	0.776	0.370	0.847	0.570	-0.137
Wages	0.636	0.178	0.147	0.866	0.568	0.608
Wage Bill	0.914	0.860	0.446	0.982	1.000	0.412
Labor Share	0.509	-0.428	0.117	0.021	-0.857	0.218

Note: This table presents the macroeconomic moments for the simple model's first subperiod calibration and compares them to the data. All figures are at the annual level. The variable g_x denotes the growth rate of x. Columns (1) to (3) present the relative growth volatility to private GDP growth volatility, correlation with private GDP growth, and one year autocorrelation in the data, respectively. Columns (4) to (6) show the same but for the model. The data are from 1979 to 2020, unless otherwise specified. Private GDP growth volatility is 4.49% from 1945 to 2020. The output growth volatility in this calibration is $\sigma(g_y) = 4.40\%$.

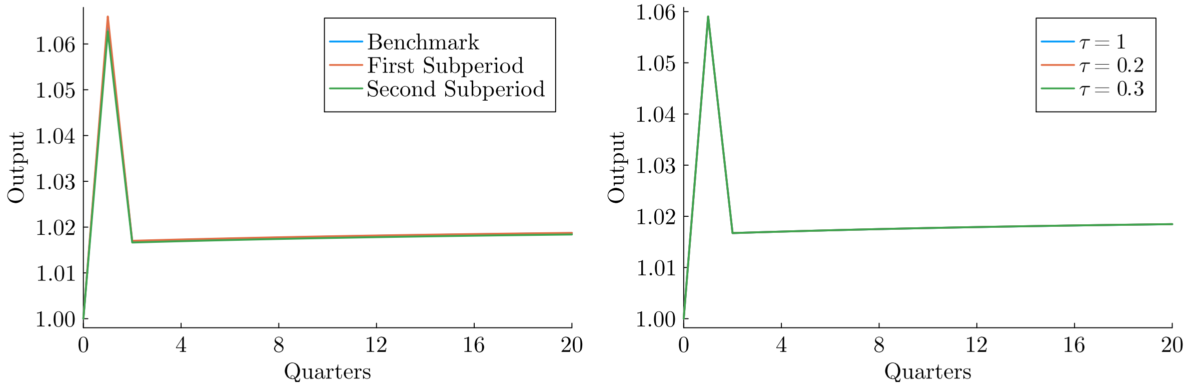
Table A.7: Simple Model Second Subperiod Calibration Macroeconomic Moments

	Data			Model		
	$\sigma(g_x)/\sigma(g_y)$ (1)	$\rho(g_x, g_y)$ (2)	AC(g_x) (3)	$\sigma(g_x)/\sigma(g_y)$ (4)	$\rho(g_x, g_y)$ (5)	AC(g_x) (6)
GDP	1.000	1.000	0.230	1.000	1.000	0.405
Consumption	0.767	0.876	0.412	0.988	1.000	0.411
Investment	2.107	0.691	0.303	2.091	0.944	0.252
Employment	0.847	0.776	0.370	0.847	0.570	-0.137
Wages	0.636	0.178	0.147	0.865	0.568	0.607
Wage Bill	0.914	0.860	0.446	0.982	1.000	0.412
Labor Share	0.509	-0.428	0.117	0.021	-0.857	0.218

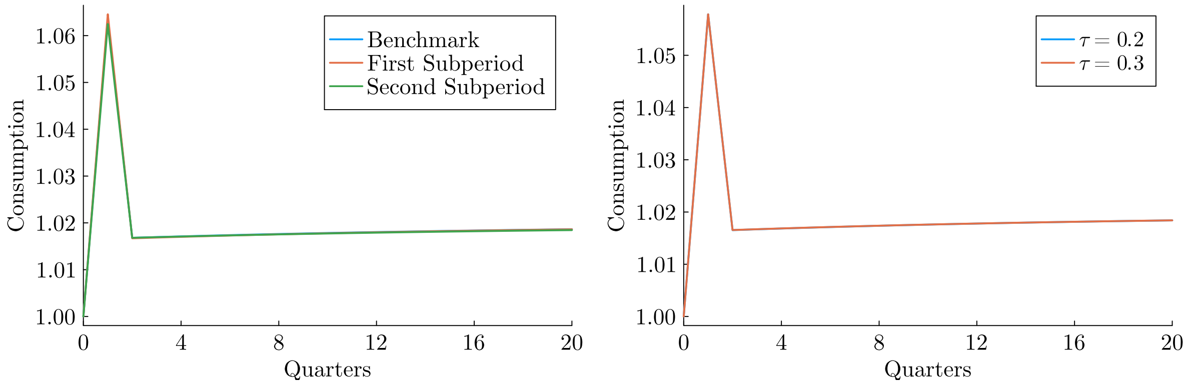
Note: This table presents the macroeconomic moments for the simple model's second subperiod calibration and compares them to the data. All figures are at the annual level. The variable g_x denotes the growth rate of x. Columns (1) to (3) present the relative growth volatility to private GDP growth volatility, correlation with private GDP growth, and one year autocorrelation in the data, respectively. Columns (4) to (6) show the same but for the model. The data are from 1979 to 2020, unless otherwise specified. Private GDP growth volatility is 4.49% from 1945 to 2020. The output growth volatility in this calibration is $\sigma(g_y) = 4.40\%$.

Figure A.1: Impulse Response Functions – Output and Consumption (All Calibrations)

(a) Output (Baseline Model) (b) Output (Simple Model)

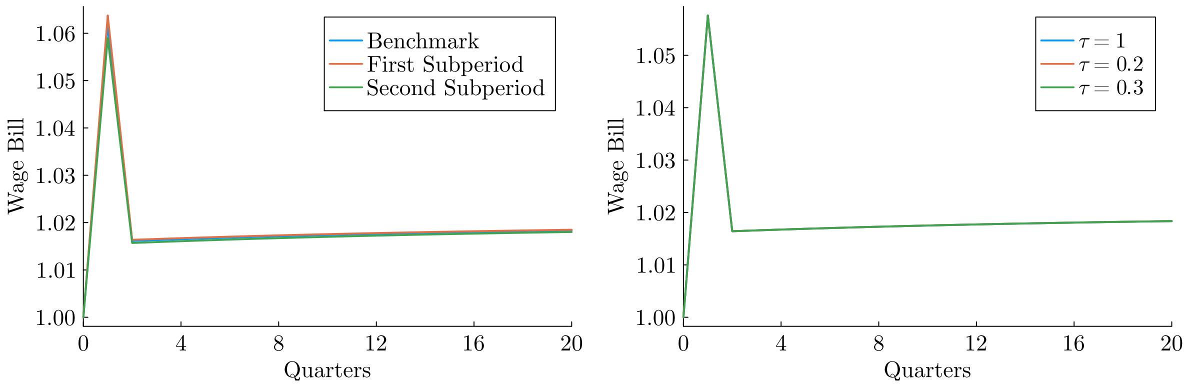


(c) Consumption (Baseline Model) (d) Consumption (Simple Model)



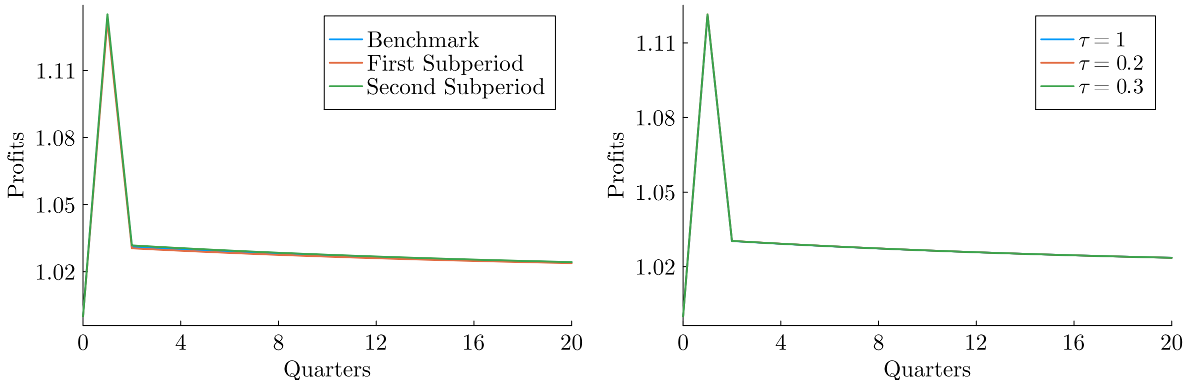
Note: This figure presents the impulse response functions for both the baseline model and simple model for output and consumption for all three calibrations. The blue line represents the benchmark calibration (labelled $\tau = 1$ for the simple model), the red line represents the first subperiod calibration (labelled $\tau = 0.2$ for the simple model), and the green line represents the second subperiod calibration (labelled $\tau = 0.3$ for the simple model). The figure is otherwise formatted similarly to Figure 6.

Figure A.2: Impulse Response Functions – Wage Bill, Profits, and Dividends (All Calibrations)



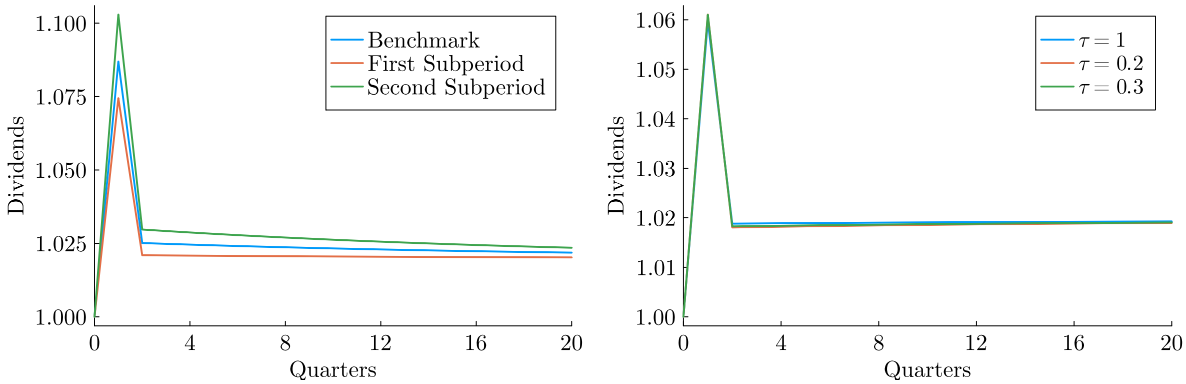
(a) Wage Bill (Baseline Model)

(b) Wage Bill (Simple Model)



(c) Profits (Baseline Model)

(d) Profits (Simple Model)

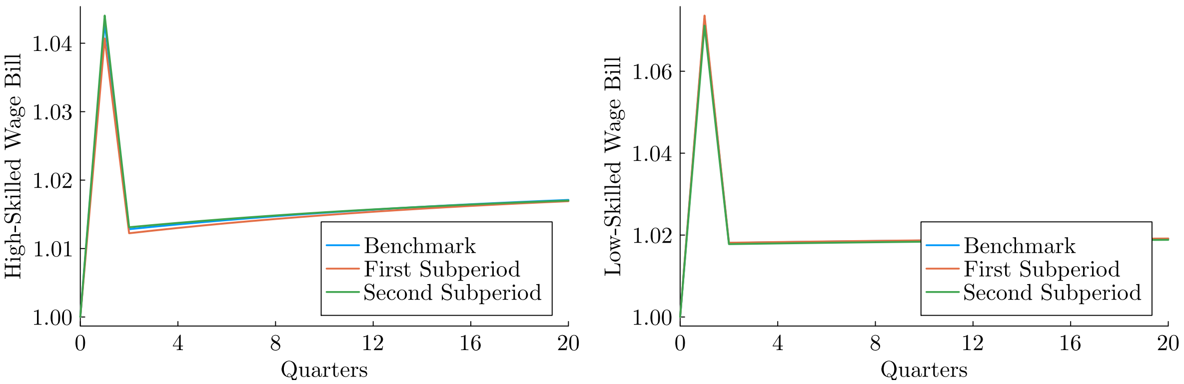


(e) Dividends (Baseline Model)

(f) Dividends (Simple Model)

Note: This figure presents the impulse response functions for both the baseline model and simple model for the wage bill, profits, and dividends for all three calibrations. The blue line represents the benchmark calibration (labelled $\tau = 1$ for the simple model), the red line represents the first subperiod calibration (labelled $\tau = 0.2$ for the simple model), and the green line represents the second subperiod calibration (labelled $\tau = 0.3$ for the simple model). The figure is otherwise formatted similarly to Figure 7.

Figure A.3: Impulse Response Functions – High-Skilled Wage Bill and Low-Skilled Wage Bill (All Calibrations)


(a) High-Skilled Wage Bill

(b) Low-Skilled Wage Bill

Note: This figure presents the impulse response functions for for the high-skilled wage bill and low-skilled wage bill for the baseline model only for all three calibrations. The blue line represents the benchmark calibration, the red line represents the first subperiod calibration, and the green line represents the second subperiod calibration. The figure is otherwise formatted similarly to Figure 9.

B Data

This section goes into greater detail on how certain variables are constructed from the data. The data from the SCF has the most intensive cleaning and merging process followed by the CPS data. This section also details the sources of the data. All the data are publicly available and can be obtained from FRED, WRDS, SPF, and the NBER.

The BEA macroeconomic time series data, BLS employment and CPI data, and FOF data can be obtained from FRED. I utilize R to automatically obtain the various data. Other languages such as Python also feature packages that enable automatic data extraction from FRED. I get data from BEA NIPA Tables 1.1.5 and 2.1. From Table 1.1.5, I take the following items denoted by their series IDs on FRED: GDP, PCEC (personal consumption), NETEXP (net exports), GCE (government expenditures), GPDI (gross private investment), PNFI (non-residential investment), and CBI (inventories). I define private GDP as GDP less government expenditures. Investment in my model corresponds to non-residential investment or equivalently total investment less residential investment and inventories. These data are taken from the start of 1947 to the end of 2021 at the quarterly frequency.

From Table 2.1, I take proprietary income (A045RC1Q027SBEA), wages to the private sector (A132RC1Q027SBEA), and supplemental income (A038RC1Q027SBEA). The series IDs are in parentheses. These are also taken from the start of 1947 to the end of 2021 at the quarterly frequency. Finally, I also take private sector employment from 1929 to 2021. However, unlike the series from the BLS, this series is only available at an annual frequency.

The CPI and private sector employment data from the BLS have series IDs CPIAUCNS and USPRIV, respectively, on FRED. The date range is 1913 to 2021 for the CPI and 1939 to 2021 for the private sector employment. Both series are available at the monthly frequency, however, I take the quarterly frequency by taking the average over the monthly series. I annualize the CPI by averaging as well.

Finally, I get corporate payout information from the FOF. All data series start from 1946 and end at 2021 and are available at the quarterly frequency. Total equity dividends are defined as the sum of net cash dividends on equity and net equity repurchases. The FRED series IDs are, respectively, NCBCEBQ027S and BOGZ1FA106121075Q. I also retrieve interest payments on debt; the total payment to debt is the interest paid less the interest received or the net interest payment. The FRED series IDs are, respectively, BOGZ1FU106130001Q and BOGZ1FA106130101Q. The unlevered dividend is the sum of the equity dividend and net interest payment.

The SCF is provided by the Federal Reserve. The main issue to clean and merge these datasets is that the SCF's variable classification changes throughout the surveys. More extensive documentation can be found in the scripts for cleaning and merging the SCF data. The 1983 and 1986 surveys have a single variable that denotes the amount of wealth in the stock market as well as other assets. The surveys from 1989 to 2001, however, do not have one simple total measure. In addition to holdings in taxable accounts, the SCF has variables on stock holdings in retirement tax advantaged accounts such as an IRA. However, the SCF does not give exact breakdowns; it only asks for what type of assets are held. There are six responses the respondent can give: (1) CDs/bank accounts, money market; (2) stock, mutual funds; (3) bonds/similar assets; (4) combination of (1), (2), and (3); (5) combination of (2) and (3); and (6) combination of (1) and (2). If the answer is either (1) or (3) the weight for stocks is imputed to be 0 and if the answer is (2) the weight is imputed for stocks to be 1. If the answer is (4), the imputed weight for stocks is 1/3 and if the answer is either (5) or (6) the weight for stocks is imputed to be 1/2. From the 2004 to 2019 surveys the structure is similar as before but the weight to stock in retirement accounts is given, thus no imputation is required. The SCF also provides basic demographic data such as age and education; a household is considered college educated if the head of household has completed at least a four-year college degree.

The MORG is the merged version of the raw CPS files that the NBER provides. The CPS has a somewhat more straightforward cleaning process. The 1979 to 2021 surveys are used. The two variables that change throughout the years are the variable indicating if the individual works in the private sector and how education is reported. From 1994 onwards, the variable and classification system to indicate private sector workers changes. However, the classification system itself is relatively straightforward and consistent with that of 1979 to 1993. There are two variables related to educational attainment from 1979 to 1991. One variable shows the highest grade attended (measured in years) and the other is an indicator on whether the highest grade attended was completed. If the highest graded attended is completed then the value in the highest grade attended variable is used directly, if the highest graded attended is not completed then education value is the highest grade attended minus 1. I assume that 16 years completed is equivalent to completing a four-year college degree. From 1992 onwards the educational attainment measures simplify to only indicate the highest grade completed.

C Model Solution

This section describes the procedure to solve the model numerically. The first subsection details how the model is detrended so that the problem is stationary. The next subsection outlines the computational algorithm to solve the detrended or stationary version of the model. Finally, the last subsection describes how the model output are analyzed and converted from quarterly level output to annual level output.

C.1 Stationary Model

The model as specified in the main text is non-stationary and cannot be solved directly. However, the model is such that it follows a balanced growth path and can be converted into a stationary problem. The stationary version of the model can be solved numerically. The detrending procedure largely follows that of Favilukis and Lin (2016b) and Favilukis, Lin, and Zhao (2020).

Non-stationary variables such as output or consumption all grow with Z_t^η and therefore can be detrended in a straightforward manner. Almost all non-stationary variables are detrended in the following convention, let X_t be a non-stationary variable at time t then let $x_t \equiv X_t/Z_t^\eta$ be the stationary or detrended counterpart. The only non-stationary variable that does not follow this convention is capital, which has the following convention $k_t = K_t/Z_{t-1}^\eta$. This is simply to make some aspects of the code easier to write, however, capital can be detrended following the exact same convention as all other variables without affecting the solution. Note that $e^{\eta g_t} = (Z_t/Z_{t-1})^\eta$.

Following this detrending convention, the low-skilled household problem is given by

$$\begin{gathered} u_{L,t} = \max\left((1-\beta)c_{L,t}^{1-\psi^{-1}} + \beta\,\mathbb{E}_t\left[e^{\eta(1-\gamma)g_{t+1}}u_{L,t+1}^{1-\gamma}\right]^{\frac{1-\psi^{-1}}{1-\gamma}}\right)^{\frac{1}{1-\psi^{-1}}} \\ \text{s.t.} \\ c_{L,t} = w_{L,t}N_{L,t}. \end{gathered} \tag{C.1}$$

Similarly, the high-skilled household's problem is given by

$$\begin{gathered} u_{H,t} = \max\left((1-\beta)c_{H,t}^{1-\psi^{-1}} + \beta\,\mathbb{E}_t\left[e^{\eta(1-\gamma)g_{t+1}}u_{H,t+1}^{1-\gamma}\right]^{\frac{1-\psi^{-1}}{1-\gamma}}\right)^{\frac{1}{1-\psi^{-1}}} \\ \text{s.t.} \\ e^{\eta g_{t+1}}a_{H,t+1}^{*} = (a_{H,t} + d_t + w_{H,t}N_{H,t} - c_{H,t})R_{t+1}^{*}. \end{gathered} \tag{C.2}$$

The SDF resulting from the high-skilled household's problem in terms of stationary variables is given by

$$M_{t+1} = \beta e^{-\eta\gamma g_{t+1}}\left(\frac{c_{H,t+1}}{c_{H,t}}\right)^{-\psi^{-1}}\left(\frac{u_{H,t+1}}{\mathbb{E}_t\left[e^{\eta(1-\gamma)g_{t+1}}u_{H,t+1}^{1-\gamma}\right]^{(1-\gamma)^{-1}}}\right)^{\psi^{-1}-\gamma}. \tag{C.3}$$

The detrended output, profits, and dividends of an individual firm are given by

$$y_{j,t} = \bar{Z}\left(\alpha N_{L,j,t}^{\eta\sigma} + (1-\alpha)\left[\lambda\left(e^{-\eta g_t}k_{j,t}\right)^\rho + (1-\lambda)\left(\theta N_{H,j,t}\right)^{\eta\rho}\right]^{\sigma/\rho}\right)^{1/\sigma}, \tag{C.4}$$

$$\pi_{j,t} = y_{j,t} - w_{H,t}N_{H,j,t} - w_{L,t}N_{L,j,t} - \xi_t, \tag{C.5}$$

$$d_{j,t} = \pi_{j,t} - i_{j,t} - \phi(i_{j,t}, k_{j,t}), \tag{C.6}$$

where $\xi_t = \zeta e^{-\eta g_t}k_t$ is the detrended fixed operating cost and

$$\phi(i_{j,t}, k_{j,t}) = \frac{\nu}{2}\left(\frac{i_{j,t}}{k_{j,t}}e^{\eta g_t} - \delta\right)^2 e^{-\eta g_t}k_{j,t}, \tag{C.7}$$

is the detrended capital adjustment cost. The laws o motion of capital by skill are given by, respectively,

$$k_{j,t+1} = (1-\delta)e^{-\eta g_t}k_{j,t} + i_{j,t}. \tag{C.8}$$

The firm's problem is now given by

$$\begin{aligned} v_{j,t}(\omega_t) = \max_{i_{j,t},N_{H,j,t},N_{L,j,t}} d_{j,t} + \mathbb{E}_t\left[e^{\eta g_{t+1}}M_{t+1}v_{j,t+1}(\omega_{t+1})\right] \\ \text{s.t.} \\ k_{j,t+1} = (1-\delta)e^{-\eta g_t}k_{j,t} + i_{j,t}, \end{aligned} \tag{C.9}$$

where ω_t is a vector of detrended state variables given by $\omega_t = (g_t, k_t)$. Now the stationary problem has only two state variables as Z_t is removed due to the detrending process. In the original problem, conditional on having ω_t the inclusion of Z_t would indicate the scaling factor or levels of the problem.

The unlevered returns of firm j at time $t+1$ are given by

$$R_{j,t+1} = \frac{v_{j,t+1}}{v_{j,t} - d_{j,t}}e^{\eta g_{t+1}}. \tag{C.10}$$

The law of motion of firm j's market value of debt assuming no default is given by

$$b_{j,t} = \rho_B e^{-\eta g_t}b_{j,t-1} + (1-\rho_B)b^*_{j,t}, \tag{C.11}$$

where $b^*_{j,t} = \kappa(v_{j,t} - d_{j,t})$. Thus, the realized gross equity return of firm j in $t+1$ in terms of detrended variables is given by

$$R^E_{j,t+1} = \frac{v_{j,t+1}}{v_{j,t} - b_{j,t} - d_{j,t}}e^{\eta g_{t+1}} - \frac{R^B_{j,t+1}b_{j,t}}{v_{j,t} - b_{j,t} - d_{j,t}}, \tag{C.12}$$

and the equity dividend is given by

$$d^E_{j,t} = d_{j,t} + b_{j,t} - R^B_{j,t}e^{-\eta g_t}b_{j,t-1}. \tag{C.13}$$

The default condition is now

$$v_{j,t+1}e^{\eta g_{t+1}} < R^P_{j,t}b_{j,t}. \tag{C.14}$$

Finally, the promised return $R^P_{j,t}$ is determined by

$$1 = \mathbb{E}_t \left[M_{t+1} \frac{\min\left\{R^P_{j,t} b_{j,t}, v_{j,t+1} e^{\eta g_{t+1}}\right\}}{b_{j,t}} \right]. \tag{C.15}$$

C.2 Computational Method

I solve this model utilizing various iteration techniques until the model converges. The approach is similar to that of Favilukis and Lin (2016b) and Favilukis, Lin, and Zhao (2020), however, since there is no firm heterogeneity and the two types of households are representative within each type, there is no need to employ algorithms akin to that of Krusell and Smith (1998). There are three broad steps to solve this model. The first is to specify an initial set of beliefs and guess for the policy function for capital. Recall that the policy functions for labor demand are trivial given the process for labor supply. The next step is to solve for the capital policy function given these beliefs and initial guess for the capital policy function. The third step is to update the beliefs using the new policy function of capital and compare the updated beliefs with the original beliefs. If the updated beliefs are sufficiently close to the original guess the algorithm terminates if not the algorithm takes the updated beliefs and new policy function as the new guesses and repeats from step two. I define an iteration of the algorithm as the entire sequence from steps two to three; step one is an initialization step. I also describe some further steps that utilize the model outputs once the model converges.

The problem is solved with Julia. In addition to the base packages, I primarily rely on the following packages: `QuantEcon`, `Interpolations`, `JLD2`, `CSV`, and `DataFrames`. For a full list please refer to the code scripts posted. Since there are two only state variables, this model can be easily solved even on a standard laptop.

C.2.1 Step One

Let Γ denote the discretized and finite state space used to numerically solve this problem. Let Γ' denote the continuous counterpart to Γ; therefore $\Gamma \subset \Gamma'$. This is needed since throughout the process since it is generally not the case that the policy function or law of motion implies a future endogenous state value that is on the original grid even if the current state is on the grid. The beliefs must also be interpolated for a similar reason. I utilize a multivariate cubic spline interpolation technique. All model functions are assumed to be defined over Γ'. The state space is over current aggregate productivity growth g_t and current capital k_t. Aggregate productivity growth g_t is discretized following Rouwenhorst (1995) into seven states. This also implies that aggregate labor supply of both types are also discretized into seven states. The exact specification of the grids for the remaining state variables are specified in the scripts.

After the parameters and grids are initialized, I produce the first guesses of the beliefs and capital policy function. The model requires beliefs over high-skilled consumption $c_{H,t}$. I call high-skilled consumption beliefs the primitive beliefs as all other beliefs can be derived using these. While the problem specified that firms pick investment, upon substituting in the law of motion of capital this can be substituted with future capital. In this script I instead solve for the future capital policy function since that is more straightforward to code. The investment policy function can be recovered using the capital law of motion. The primitive beliefs and policy function for capital are used to generate a consistent belief for the SDF.

Let $\hat{c}^n_H(\omega)$ denote the beliefs of high-skilled consumption evaluated at state $\omega \in \Gamma'$ in the n-th iteration of the algorithm. In general in this section, the superscript n indicates the iteration. For $n = 1$, I set the initial beliefs over high-skilled consumption to be the wage bill for high-skilled labor. We know from the beginning the value of the current wage bill given the simplicity of the labor problem. The primitive beliefs are initially set over each discrete point in Γ and then interpolated so that they can be evaluated over any point in Γ'.

C.2.2 Step Two

Given the primitive beliefs and policy function guess, I can recover the beliefs of the SDF that are consistent with these. However, there is an intermediate step given that consumers have Epstein and Zin (1989) and Weil (1990) preferences. The beliefs of high-skilled consumer's utility need to be computed first. The beliefs are given by

$$\hat{u}^n_H(\omega) = \left((1-\beta) \left[\hat{c}^n_H(\omega)\right]^{1-\psi^{-1}} + \beta\, \mathbb{E}_t \left[e^{\eta(1-\gamma) g_{t+1}} \left[\hat{u}^n_H(\omega')\right]^{1-\gamma} \right]^{\frac{1-\psi^{-1}}{1-\gamma}} \right)^{\frac{1}{1-\psi^{-1}}}, \tag{C.16}$$

where $\omega \in \Gamma'$ is the current state and $\omega' \in \Gamma'$ is the future state. I denote $\mathbb{E}_t[\cdot]$ as the expectation with all information in ω. The utility over Γ' can be recovered using a simplified version of value function iteration as utility is defined recursively. However, since there is already beliefs over consumption there is no need to apply a maximum operator in the algorithm since it is assumed this consumption belief is already maximizing utility conditional on the state. This procedure mirrors the intermediate step introduced when utilizing Howard's improvement in a value function iteration algorithm. This greatly reduces the computational burden of finding the utility of high-skilled agents and reduces the value function iteration process to just looping over the discretized state space and checking for convergence.

With the utility computed, the belief of the SDF can be recovered. This is given by

$$\hat{M}^n(\omega, g') = \beta e^{-\eta\gamma g'} \left(\frac{\hat{c}^n_H(\omega')}{\hat{c}^n_H(\omega)} \right)^{-\psi^{-1}} \left(\frac{\hat{u}^n_H(\omega')}{\mathbb{E}_t \left[e^{\eta(1-\gamma) g'} \left[\hat{u}^n_H(\omega')\right]^{1-\gamma} \right]^{(1-\gamma)^{-1}}} \right)^{\psi^{-1}-\gamma}, \tag{C.17}$$

where g' is the realization of the next period's aggregate productivity growth which is a component in ω'. Recall that given the beliefs and capital policy function, the endogenous state variables for the next period are known using the corresponding law of motion or policy function.

Taking the primitive beliefs and SDF as given, the policy function that is consistent with these can be computed. This is solved using policy function iteration. Let $k^n(\cdot)$ be the policy function that is computed using the n-th iteration of beliefs.

C.2.3 Step Three

The final step of the algorithm is to update the primitive beliefs given $k^n(\cdot)$. Let $\tilde{c}_H^n(\omega)$ denote the implied beliefs over high-skilled consumption given $k^n(\cdot)$ on the n-th belief iteration evaluated at ω. Then for all $\omega \in \Gamma$, the discretized state space, compute

$$\tilde{c}_H^n(\omega) = w_H^n(\omega)N_H(\omega) + d^n(\omega), \tag{C.18}$$

For high-skilled consumption, the current average high-skilled wage and aggregate dividend are needed. The current aggregate dividend $d^n(\omega)$ is computed in a straightforward manner using the implied investment from $k^n(\cdot)$ and the various inputs that are recovered by knowing ω.

With the implied primitive beliefs computed, I compute the maximum absolute error as follows

$$\varepsilon^C = \max_{\omega \in \Gamma} \left| \hat{c}_H^n(\omega) - \tilde{c}_H^n(\omega) \right|, \tag{C.19}$$

If $\varepsilon^C < \bar{\varepsilon}$ for some threshold $\bar{\varepsilon}$ then the algorithm terminates. If not, then the algorithm updates the primitive beliefs using the implied beliefs as follows

$$\hat{c}_H^{n+1}(\omega) = \chi(n)\hat{c}_H^n(\omega) + (1 - \chi(n))\tilde{c}_H^n(\omega), \tag{C.20}$$

where $\chi(n) \in [0, 1)$ is the weight on the old belief that is a function of the iteration. A weight on the old belief is needed to ensure the algorithm's stability and a higher value can even speed up convergence at some stages. The choice of $\chi(\cdot)$ depends on the calibration and model. Generally, $\chi(\cdot)$ is given by

$$\chi(n) = \begin{cases} \chi_1 & \text{if } 1 \leq n \leq \bar{n}_1 \\ \chi_2 & \text{if } \bar{n}_1 < n \leq \bar{n}_2 \\ \chi_3 & \text{if } \bar{n}_2 < n, \end{cases} \tag{C.21}$$

where $\chi_1 > \chi_2 > \chi_3$. For now $\chi_1 \in [0.80, 0.98]$, $\chi_2 \in [0.65, 0.85]$, and $\chi_3 \in [0.40, 0.67]$. For now I use cutoffs $\bar{n}_1 \in \{25, 35\}$ and $\bar{n}_2 \in \{40, 55, 60\}$. It may be possible for the algorithm to be solved more quickly by using lower weights and thresholds. Once the primitive beliefs are updated these are used as the primitive beliefs for iteration $n + 1$, the initial policy function guess is $k^n(\cdot)$ and the algorithm returns to the beginning of step two.

C.2.4 Model Simulation

Once the algorithm converges and terminates, I recover the value function and simulate the economy by generating a path for aggregate productivity growth and specifying an initial condition for aggregate capital. The value function is needed since I compute returns. The level of the value function has meaning since it is corresponding to a firm's value maximization problem. Value function iteration using the solved policy function recovers the value function. Similar to computing the utility, the value function iteration here does not need to use a maximum operator since the policy function already solves the problem and thus this procedure is much quicker.

With value function the solved, I proceed to the simulation of the economy. Here is also when I introduce the firm's capital structure. Since the economy satisfies the Modigliani and Miller (1958) conditions, it is relatively straightforward to introduce an exogenous capital structure and can be done after solving for the policy and value functions. Newton's method is used to find the promised return on corporate debt. I assume that the economy's firm starts with zero debt. The economy is simulated for a total of $T = 30,000$ periods where the first $T_b = 10,000$ periods are used as a burn-in and the last $T - T_b$ periods are kept to minimize the influence of the initial conditions. The simulated output is analyzed in the same manner as the data at both the quarterly and annual level. The following subsection discusses how the initial quarterly level output is converted into annual level output.

Both the time series of the primitive beliefs and the actual value of the corresponding variables are created and compared as an accuracy check. I compute the absolute relative error for high-skilled consumption (the sole variable that has a primitive belief) across all t, which follows a somewhat similar logic as den Haan (2010). Let the absolute relative error at time t for high-skilled consumption be defined as

$$\mathcal{E}_t^C = \left| \frac{\tilde{c}_{H,t} - \hat{c}_{H,t}}{\hat{c}_{H,t}} \right|. \tag{C.22}$$

I suppress the iteration superscript and function notation in equation (C.22) for ease of exposition. To evaluate the accuracy I take the mean, median, and maximum across all t for all primitive beliefs. I provide summary statistics of $\mathcal{E}_t^C$ across all models in Table C.1. The maximum relative absolute error across all models are all very small indicating that the algorithm is able to accurately solve the model.

Table C.1: Relative Absolute Error Across Models and Calibrations

	Mean (1)	Min (2)	P25 (3)	Median (4)	P75 (5)	Max (6)
Baseline Model (Benchmark)	4.89e-06	7.21e-10	3.39e-06	4.89e-06	6.46e-06	1.20e-05
Baseline Model (First Subperiod)	6.03e-06	8.34e-10	4.27e-06	6.10e-06	7.86e-06	1.38e-05
Baseline Model (Second Subperiod)	2.97e-06	2.55e-11	1.98e-06	2.99e-06	3.96e-06	1.36e-05
Simple Model (Benchmark)	8.44e-07	1.48e-10	5.59e-07	8.53e-07	1.13e-06	2.06e-06
Simple Model (First Subperiod)	4.49e-06	5.55e-07	3.88e-06	4.59e-06	5.18e-06	6.39e-06
Simple Model (Second Subperiod)	2.05e-06	3.80e-10	1.31e-06	2.04e-06	2.78e-06	5.35e-06

Note: This table shows the summary statistics of the relative absolute errors across all models and calibrations. Column (1) shows the mean relative absolute error. Columns (2) to (6) show the minimum, 25th percentile, median, 75th percentile, and maximum relative absolute error. All figures are in percentage points.

A similar simulation approach is utilized for computation of the impulse response functions. While the economy starts with the identical initial condition for aggregate capital as for the simulations for computing the moments, the aggregate productivity growth time series is always set to the middle point on the discretized grid. This approach approximates a stochastic steady state for the stationary problem. The economy is then run for 20,000 periods to allow the economy to converge to the stochastic steady state capital level. Once this is finished, an additional 100 periods are run for two counterfactuals. The first counterfactual is a continuation of the stochastic steady state for 100 periods. For the second counterfactual, the path of productivity growth jumps to a higher productivity growth for four periods before returning to the middle point for the remainder of the periods, which reflects a positive and temporary productivity growth shock. Since the aggregate productivity growth process is discretized into seven states, the productivity growth shock takes the economy to the second highest productivity growth level from the middle point. I use both time series to create the impulse reponse function figures.

C.2.5 Computing Prices and Returns of Dividend Strips

The price of dividend strips and the corresponding expected and realized returns can be computed using the policy and value functions. Let $D_t^{(n)}$ be the price of the dividend strip that entitles the owner to the dividend paid n periods in the future at period t. The price of the first dividend strip is given by

$$D_t^{(1)} = \mathbb{E}_t \left[M_{t+1} D_{t+1} \right]. \tag{C.23}$$

The remainder of the term structure is defined recursively. For $n \geq 2$, we have

$$D_t^{(n)} = \mathbb{E}_t \left[M_{t+1} D_{t+1}^{(n-1)} \right]. \tag{C.24}$$

The realized gross returns are then given by

$$R_{t+1}^{D,(n)} = \frac{D_{t+1}^{(n-1)}}{D_t^{(n)}}, \tag{C.25}$$

for all $n \geq 1$ and defining $D_{t+1}^{(0)} \equiv D_{t+1}$.

Following the same notation convention and procedure to detrend these variables we get

$$d_t^{(n)} = \mathbb{E}_t[M_{t+1} e^{\eta g_{t+1}} d_{t+1}^{(n-1)}] \tag{C.26}$$

$$R_{t+1}^{D,(n)} = \frac{d_{t+1}^{(n-1)} e^{\eta g_{t+1}}}{d_t^{(n)}}. \tag{C.27}$$

Given the capital policy function, the dividend in equilibrium is a function of the state variables Ω_t (ω_t after detrending). Therefore, the price of the entire dividend term structure can be computed recursively over the state space, which in turn allows me to recover the returns and return volatilities.

I separately simulate the prices and returns of the term structure of dividends given the solved policy and value functions for 30,000 periods with 10,000 periods used as a burn-in. The expected returns and return volatility for the first 200 quarters of the dividend term structure are computed for both the baseline and simple model across all calibrations.

C.3 Annual Aggregation

The model is solved and calibrated at the quarterly level. However, annual moments and results are also produced. This subsection outlines how the quarterly level model output are converted to annual level output. Let $G_{x,t} = X_t / X_{t-1}$ be the gross growth rate of non-stationary variable X_t at time t. In terms of stationary variables we have

$$G_{x,t} = \frac{X_t}{X_{t-1}} = \frac{x_t Z_t^{\eta}}{x_{t-1} Z_{t-1}^{\eta}} = \frac{x_t}{x_{t-1}} e^{\eta g_t}. \tag{C.28}$$

This expression is used to compute quarterly growth rates from output from the stationary model.

Now consider the problem of aggregating quarterly level data to the annual level. In the data for figures such as GDP or investment, the annual data is just the sum of the quarterly data (if the quarterly data are in annualized terms then the annual figure is the mean). From here on I use the following subscript convention for some variable X, $X_{t,q}$ is the value of X at year t quarter q. Thus, $X_{t,q}$ is a quarterly variable. Let X_t be the annual counterpart. Thus,

$$X_t = \sum_{q=1}^{4} X_{t,q}. \tag{C.29}$$

In terms of the stationary model's outputs we get

$$X_t = \sum_{q=1}^{4} x_{t,q} Z_{t,q}^{\eta}. \tag{C.30}$$

I use the convention that I detrend the annual level variables with the first quarter of the year's aggregate productivity factor, thus $x_t = X_t Z_{t,1}^{-\eta}$. Therefore,

$$x_t = \sum_{q=1}^{4} x_{t,q} \left(\frac{Z_{t,q}}{Z_{t,1}} \right)^{\eta} = x_{t,1} + \sum_{q=2}^{4} x_{t,q} e^{\eta \sum_{r=2}^{q} g_{t,r}}. \tag{C.31}$$

The annual gross growth rate then, is given by

$$G_{x,t} = \frac{X_t}{X_{t-1}} = \frac{x_t}{x_{t-1}} \left(\frac{Z_{t,1}}{Z_{t-1,1}} \right)^{\eta} = \frac{x_t}{x_{t-1}} e^{\eta (g_{t-1,2} + g_{t-1,3} + g_{t-1,4} + g_{t,1})}. \tag{C.32}$$

D Additional Models

D.1 Simple Model

This section provides more details of the simple model presented in the main text. The notation largely follows that of the original model but is slightly adjusted since households are not differentiated by skill and only by financial market participation. Unconstrained households have access to financial markets and have mass $\tau \in (0, 1]$ and constrained households are hand-to-mouth have mass $1 - \tau$. Let P index unconstrained households and X index constrained households. The unconstrained household's problem is given by

$$\begin{gathered} U_{P,t} = \max \left((1-\beta) C_{P,t}^{1-\psi^{-1}} + \beta \, \mathbb{E}_t \left[U_{P,t+1}^{1-\gamma} \right]^{\frac{1-\psi^{-1}}{1-\gamma}} \right)^{\frac{1}{1-\psi^{-1}}} \\ \text{s.t.} \\ A_{P,t+1}^{*} = (A_{P,t} + W_t \tau N_t + D_{P,t} - C_{P,t}) R_{P,t+1}. \end{gathered} \tag{D.1}$$

The constrained households consume $C_{X,t} = W_t (1 - \tau) N_t$. The risk-free asset is in net-zero supply and all corporate securities have one unit of supply. Since there is are representative households within type, in equilibrium all unconstrained households always hold no risk-free assets and the value-weighted market portfolio of corporate securities. All households inelastically supply labor N_t, which is an exact function of current aggregate productivity growth. Since I model both unconstrained and constrained households as having the same labor process (adjusted for their weights), it is equivalent to model them as an aggregate and as separate components that are exactly related.

There is a unit mass of identical firms indexed by $j \in [0,1]$. Firms' production technology is given by

$$Y_{j,t} \equiv \bar{Z}\left(\alpha K_{j,t}^{\sigma} + (1-\alpha)(Z_t N_{j,t})^{\eta\sigma}\right)^{1/\sigma}, \tag{D.2}$$

where α is the capital intensity of production and σ governs the elasticity of substitution of capital and labor. The elasticity of substitution is given by $1/(1-\sigma)$. The firm's profits and dividends are given by, respectively,

$$\Pi_{j,t} \equiv Y_{j,t} - W_t N_{j,t} - \Xi_t \tag{D.3}$$

$$D_{j,t} \equiv \Pi_{j,t} - I_{j,t} - \Phi(I_{j,t}, K_{j,t}). \tag{D.4}$$

The fixed operating cost Ξ_t and capital adjustment costs $\Phi(I_{j,t}, K_{j,t})$ are defined identically as before. Aggregate productivity growth is given by $g_t = \ln(Z_t/Z_{t-1})$ and follows the same stationary AR(1) process as in the original model. The law of motion for capital are also the same as before. Thus, the firm's problem is given by

$$\begin{gathered} V_{j,t}(\Omega_t) = \max_{I_{j,t},N_{j,t}} D_{j,t} + \mathbb{E}_t\left[M_{t+1}V_{j,t+1}(\Omega_{t+1})\right] \\ \text{s.t.} \\ K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t}. \end{gathered} \tag{D.5}$$

The vector of state variables Ω_t is given by $\Omega_t = (Z_t, g_t, K_t)$. The SDF M_{t+1} is derived from the unconstrained household's problem. The firm has the same capital structure properties as before. The aggregation and equilibrium conditions follow analogously from the original model. Finally, solving this model quantitatively follows the same detrending and numerical procedure shown in Appendix C. I set $\chi_1 = 0.8$, $\chi_2 = 0.6$, $\chi_3 = 0.4$, $\bar{n}_1 = 25$, and $\bar{n}_2 = 40$ for the simple model for all calibrations.

D.2 Exchange Model

This section outlines the exchange model, which is similar to the simple model presented in the previous section, however, the dividend process here is exogenous. I use the same notation for the exchange model as for the simple model. Both types of households are endowed with an exogenous income process that mirrors their labor income, denoted by $W_t N_t$. Unconstrained households that participate in financial markets also can trade shares of the representative firm and the risk-free asset. There is mass of $\tau \in (0,1]$ unconstrained households. Both households solve the same problem as before, and as a result equilibrium consumption at each period t for both households are given by

$$
\begin{aligned}
C_{P,t} &= \tau W_t N_t + D_t \\
C_{X,t} &= (1 - \tau) W_t N_t .
\end{aligned}
$$

In order to compare the exchange model with the simple model, I average wage and labor supply values conditional on some value of capital $\bar{K}$. I set $\bar{K}$ to the grid point on the capital grid that is closest to the mean level of capital in the simulations for the simple model's benchmark calibration. Furthermore, I also take the dividend values over Γ from this model to generate the exogenous dividend process. Note that since the dividend process in this model is exogenous, changes in the SDF does not change the dividend process. Then with these objects, I take the capital policy function from the baseline calibration of the simple model to simulate a path for K_t given some starting value K_0 and exogenous path for Z_t to generate a simulation for the dividend process. This approach allows me to simply compute the comparative statics of the exchange model with respect to τ while still maintaining comparability to the simple model. I apply a simple leverage adjustment to convert the unlevered returns to levered equity returns in the exchange model; the unlevered net returns are multiplied by a factor of 5/4.[18]

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[18] This implies that the average firm is financed with 40% debt and 60% equity, which is consistent with the data and is a similar approach Boldrin, Christiano, and Fisher (2001) and Papanikolaou (2011) utilize to adjust unlevered returns.